A Problem-Solving Approach to College Algebra

Marcel B. Finan
Arkansas Tech University

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PREFACE

This book is a non traditional textbook in college algebra. Its primary objective is to encourage students to learn from asking questions rather than reading a detailed explanations of the material discussed in a typical textbook. As a result, a critical prerequisite for students wishing to take this course is attendance. Supplemental information of the mathematics involved in this book will be provided after the problems have been read and questions have been posed. Another main objective of this book is for the students to gain confidence in their ability to use mathematics. A final objective is to prepare students with a solid foundation for subsequent courses in mathematics and other disciplines.

Marcel B. Finan
Russellville, Arkansas
May 2002.

Contents

Chapter 1

Fundamental Concepts

In this chapter we consider the basic topics that every student needs to be familiar with when taking college algebra, namely, the familiarity with the sets of numbers and their properties, the question of solving equations, the skill of applying algebra for solving real world problems, and most importantly the concept of a function.

1.1 Sets of Numbers and their Properties

In what follows, by an **integer** we mean a number in the collection

$$\{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$$

Exercise 1
A **rational** *number is a number that can be represented by a fraction of the form $\frac{a}{b}$ where a and b are both integers with $b \neq 0$. Decimal notation for rational numbers either terminates or repeats. A number that is not rational is called* **irrational**. *Consider the following numbers:* $0, -\sqrt{3}, \frac{2}{3}, 0.\overline{45}, -\frac{\pi}{2}, \pi$.

(i) Which are rational numbers?
(ii) Which are irrational numbers?

Exercise 2
Write the number $0.125125125\cdots$ *as a fraction. Thus, convince yourself that a decimal numbers with repeating digits is a rational number.*

Exercise 3
(i) Find a number that is rational but not an integer.
(ii) Find a real number that is not rational.
(iii) Find a real number that is not irrational.

Exercise 4
To find the sum of two numbers with opposite signs, ignore the negative sign, subtract the smaller number from the larger number. The sum has the same sign as the sign of the larger number. For example, to find $-6+13$ *we subtract* 6 *from* 13 *to obtain* 7 *and since the larger number* 13 *has a positive sign then* $-6+13=7$. *Calculate* $-35.4+2.51$.

Exercise 5
State the property given by each sentence, where a, b, *and* c *are real numbers:*

(a) $a+b=b+a$
(b) $a \cdot b=b \cdot a$
(c) $a+(b+c)=(a+b)+c$
(d) $a \cdot (b \cdot c)=(a \cdot b) \cdot c$
(e) $a+0=0+a$
(f) $a \cdot 1=1 \cdot a$
(g) $a+(-a)=(-a)+a=0$
(h) $a \cdot \frac{1}{a}=\frac{1}{a} \cdot a=1, \; a \neq 0$
(i) $a(b+c)=ab+ac$

Exercise 6
The opposite of the opposite of a *is just* a, *that is,* $-(-a)=a$. *Calculate* $-3.6-(-7)$.

Exercise 7
The product (or ratio) of two positive numbers is always positive; the product (or ratio) of two negative numbers is always negative. Calculate $(1-6)(-3)$.

Exercise 8
The product of two fractions $\frac{a}{b}$ *and* $\frac{c}{d}$ *is defined by* $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. *Simplify:* $(-6\frac{3}{7})(\frac{14}{-5})$.

Exercise 9
The sum or difference of two fractions: $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$. *Calculate:* $(3\frac{1}{3}-5\frac{2}{3})(-\frac{9}{2})$.

Exercise 10
The notations $a \div b$ *or* $\frac{a}{b}$ *mean that there exists a* **unique** *number* c *such that* $c \times b = a$. *Find the value, if possible, of the ratio* $(-3) \div 0$.

Exercise 11
To add fractions with non identical denominators, one finds the **least common denominator**, *that is the least number divisible evenly by all the denominators. This is also known as the* **least common multiple.** *For example, to find the LCM of two numbers* a *and* b *we first write each number as a product of the form* $a = 2^{c_1}3^{c_2}5^{c_3}\cdots$ *and* $b = 2^{d_1}3^{d_2}5^{d_3}\cdots$. *Then* $LCM(a,b) = 2^{max(c_1,d_1)}3^{max(c_2,d_2)}5^{max(c_3,d_3)}\ldots$
Find $LCM(24, 126)$.

Exercise 12
Find the value of

$$\frac{3}{5} + \frac{2}{7} =$$

Exercise 13
Find the value of

$$-8 - (-8) =$$

Exercise 14
Find the sum

$$\frac{x}{3} + \frac{2x}{5} =$$

Exercise 15
Find the product

$$\frac{3x}{4y}\frac{8y^2}{x^2} =$$

Exercise 16
Find the value of

$$(-5)(-1)(-7) =$$

Exercise 17
The distributive law of multiplication states $a(b+c) = ab+ac$ *or* $(a+b)c = ac+bc$. *Multiply*

$$x(x - 2y) =$$

Exercise 18
Multiply

$$(2x - 3y)(x + 2y) =$$

Exercise 19
The world's fastest airliner, the Concorde, travels one mile in 0.0006897 *hour and carries 128 passengers. Find its rate in miles per hour.*

Exercise 20
If 720 *is* 0.6% *of* x, *what is* x?

Exercise 21
What number is obtained from increasing 300 *by* 115%*?*

Exercise 22
If an amount of money P *(called principal) is invested at an annual simple rate* r *for a period of* t *years then the balance in the account at the end of* t *years is given by the formula* $A = P + I = P + Prt = P(1 + rt)$.
What amount must be invested at 8% *simple interest so that* \$50 *interest is earned at the end of* 6 *months?*

Exercise 23
What is the interest on \$500 *invested at* 9% *per year simple interest for 4 years?*

Exercise 24
The set of real numbers, denoted by $\mathbb{R}$*, is modeled using a line directed to the right. Any point* A *on this line is associated with a real number* a *called the* **abscissa** *of* a*. We write* $A(a)$*. Plot the points* $A(-3), O(0), B(2.5),$ *and* $C(\sqrt{2})$*.*

Exercise 25
If $x \geq 0$ *then the* **absolute value** *of* x *is* $|x| = x$*; if* $x < 0$ *then* $|x| = -x$*. Geometrically, the absolute value of a number* a *is its distance from* 0 *on the number line. Calculate* $\frac{-4}{|-4-1|}$*.*

Exercise 26
The distance between two points $A(a)$ *and* $B(b)$ *on the real line is given by the formula* $d(A, B) = |a - b|$*. Find the distance between the points* $A(6\frac{1}{2})$ *and* $B(4.2)$*.*

Exercise 27
When simplifying algebraic expressions one uses the following rules for order of operations:

(i) Do all calculations within grouping symbols before operations outside. When nested grouping symbols are present work from inside out.
(ii) Evaluate all exponential expressions.
(iii) Do all multiplications and divisions in order from left to right.
(iv) Do all additions and subtractions in order from left to right.

Rewrite the following expression without using parentheses.

$$4\{-[-(a-b) + 2(b-a)] - (a+b)\}$$

Exercise 28
Rewrite the following expression without using parentheses.

$$-[-2(a+c) - (b-c) + 3(-a+c)]$$

1.2 Solving Equations

An **equation** is an equality between two algebraic expressions. A number that satisfies an equation is called a **solution** or a **root.**

Exercise 29
Which of the following numbers is a solution to the equation

$$3x + 7 = -5$$

(A) -4 *(B)* -7 *(C)* 4 *(D)* $-\frac{5}{3}$ *(E) None of the above.*

The collection of all solutions to an equation is called the **solution set.** To solve an equation is to find the solution set.

Four important principles for solving equations:

(1) The Addition Principle: Adding or subtracting the same number to both sides of an equation does not change the solution set of the original equation.

(2) The Multiplication Principle: Multiplying or dividing both sides of an equation by the same number does not change the solution set of the original equation.

(3) The Principle of Zero Products: If $ab = 0$ then $a = 0$ or $b = 0$.

(4) The Principle of Square Roots: If $x^2 = k$ with $k \geq 0$ then $x = \pm\sqrt{k}$.

Exercise 30
Solve

$$2x - 5 = 3x + 7$$

Exercise 31
Sometimes to solve an equation, you must simplify the equation before using the properties listed above. Solve

$$(x + 2)(x + 3) = x^2 + 16$$

Exercise 32
When an equation involves fractions, you can eliminate the fractions by multiplying both sides of the equation by the least common denominator. Solve

$$\frac{1}{x} - \frac{x}{5} = \frac{1 - 3x}{15}$$

Exercise 33
Solve

$$\frac{x}{2} + \frac{2x - 1}{4} = \frac{x + 1}{8}$$

Exercise 34
Solve

$$\frac{-2}{x^2 - 1} + \frac{x}{x - 1} = 0$$

Exercise 35
Solve

$$\frac{2x + 1}{x(2x + 3)} + \frac{1}{2x} = \frac{3}{2x + 3}$$

Exercise 36
In a product ab*, we call* a *and* b *factors. An important property of numbers is that, if* $ab = 0$ *then either* $a = 0$ *or* $b = 0$*. This is known, as the zero product property. Solve*

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = 0$$

Exercise 37
Solve

$$\frac{3x}{x-1} + 2 = \frac{3}{x-1}.$$

Exercise 38
Solve

$$3x - 5 = 4$$

Exercise 39
Solve

$$x^3 = 25x$$

Exercise 40
Solve

$$\frac{1}{x} + \frac{1}{x-1} = \frac{1}{x(x-1)}.$$

1.3 Applied Problems

Applied problems are also known as word problems or mathematical models. The following is one of the strategies that can be used for solving word problems:

(1) Read the problem several times until you completely understand it.
(2) If possible, draw a diagram to illustrate the problem.
(3) Choose a variable and write down what it represents.
(4) Represent any other unknowns in terms of that variable.
(5) Write an equation that models the situation.
(6) Solve the equation.
(7) Check your answer by using it to solve the original problem (and not the equation).

Exercise 41
Beth grossed \$435 *one week by working 52 hours. Her employer payes time-and-a-half for all hours worked in excess of 40 hours. With this information, can you determine Beth's regular hourly wage?*

Exercise 42
Judy, an investor with \$70,000*, decides to place part of her money in corporate bonds paying* 12% *per year and the rest in a Certificate of Deposit paying* 8% *per year. If she wishes to obtain an overall return of* 9% *per year, how much should she place in the CD investment?*

Exercise 43
In a chemistry laboratory the concentration of one solution is 10% *of HCl and that of a second solution is* 60% *HCl. How many millilitres (mL) of the* 10% *solution should be used to obtain 50 mL of a* 30% *HCl solution?*

Exercise 44
From each corner of a square piece of sheet metal, remove a square of side 9 *centimeters. Turn up the edges to form an open box. If the box is to be hold 144 cubic centimeters, what should be the dimensions of the piece of sheet metal?*

Exercise 45
A metra commuter train leaves Union Station in Chicago at 12 noon. Two hours later, an Amtrak train leaves on the same track, travelling at an average speed that is 50 miles faster than the Metra train. At 3 p.m., the Amtrak train is 10 miles behind the Metra train. How fast is the Metra train?

Exercise 46
The sum of three consecutive even integers is 72. Find the integers.

Exercise 47
If two edges of a cube are increased by 2 cm and the remaining edge is increased by 1 cm, the volume of the resulting rectangular box is $\frac{29}{5}$ *cm*^{3} *larger than the volume of the original cube. Find the length of an edge of the original cube.*

Exercise 48
A flask has 14 ounces of a certain chemical known to contain 20% *alcohol. How many ounces of pure alcohol must be added in order to raise the concentration to* 40%.
Hint: $\frac{\text{Amount of alcohol in new solution}}{\text{Total amount of liquid in the new solution}} = \frac{40}{100}$.

Exercise 49
Suppose a retailer runs a sale in which each item is discounted 20%. *If* x *is the price of an item before the sale, give an algebraic expression for the sale price.*

Exercise 50
You receive a monthly salary of \$2,000 *plus a commission of* 10% *of sales. Suppose your monthly sale is* \$1,480 *then what is your wage for the month?*

Exercise 51
The sum of two numbers is 8 *and their product is* 5. *What is the sum of their squares?*

Exercise 52
An auto repair shop charges \$20 *shop charge plus* \$25 *per hour for labor. If the total charge for a repair job is* \$80 *plus parts, how many hours of labor did the job require?*

Exercise 53
A comparison shopper notes that the competition runs a sale in which a coat is marked down 20% *to* \$72. *What was the original price?*

1.4 The graph of an Equation

By the **graph** of an equation we mean the collection of all ordered pairs (x, y) that satisfy the equation. Recall that an ordered pair is associated to a point in the Cartesian system.

Exercise 54 *(Distance Formula)*
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points in Cartesian plane then the distance between them is found by the formula $d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Find the distance between $(-1, 2)$ and $(3, 5)$.

Exercise 55
Suppose that (x, y) is equidistant from the points $(1, 3)$ and $(-1, 2)$. Find a relationship between x and y.

Exercise 56 *(Midpoint Formulas)*
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points in the Cartesian plane then the midpoint $M(x, y)$ is given by $x = \frac{x_1+x_2}{2}$ and $y = \frac{y_1+y_2}{2}$. Find the midpoint of the segment determined by the points $(\frac{3}{4}, \frac{7}{8})$ and $(-\frac{1}{7}, 4)$.

Exercise 57
Find all numbers x for which the distance between $(-1, x)$ and $(2, 0)$ is 5.

Exercise 58
Suppose that (x, y) is on the perpendicular bisector of $(4, 5)$ and $(3, -2)$. Find an equation giving the relationship between x and y.

Exercise 59
Suppose that a triangle with vertices $(x, y), (2, 3)$ and $(5, -9)$ is equilateral. Find an equation giving the relationship between x and y.

Exercise 60
Graph the equation $3x - 4y + 12 = 0$.

Exercise 61
Find the distance between $(7, 4)$ and $(-2, 1)$.

Exercise 62
The equation of a circle with center (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$. Find the center of the circle $(x - 2)^2 + (y + 3)^2 = 9$.

Exercise 63
Find an equation of the circle centered at $(3, -2)$ and with radius 3.

Exercise 64
Find an equation of the circle centered at $(-4, 6)$ and passing through the point $(-1, 2)$.

Exercise 65
Find the center of the circle: $x^2 + y^2 + 4x - 6y + 12 = 0$.

Exercise 66
Graph the equation $y = \frac{1}{x}$.

Exercise 67
Graph the equation $y = x^2$.

Exercise 68
Graph the equation $y = |x|$.

Exercise 69
Graph the equation $y = \sqrt{x}$ *for* $x \geq 0$.

Exercise 70
Graph the equation $y = x^3$.

Exercise 71
Graph the equation $x = y^2$.

Exercise 72
Find an equation for the circle whose center is $(2, -3)$ *and whose radius is equal to 4.*

Exercise 73
Find an equation for the circle whose center is $(-\frac{1}{2}, \sqrt{2})$ *and whose radius is equal to* $2\sqrt{2}$.

Exercise 74
Find the center and the radius of the circle:$x^2 + y^2 + 2x - 6y = 15$.

Exercise 75
Find an equation for the set of all points (x, y) *with the property that the sum of the distances from* (x, y) *to* $(1, 0)$ *and from* (x, y) *to* $(-1, 0)$ *is equal to 6.*

Exercise 76
Graph the equation $y = x^2 + 2$.

1.5 The Concept of a Function

A function f with a source set (or **domain**) A is a rule which assigns to every input value x of A a unique output value y. We write $y = f(x)$. We call y the **image** of x under f. The collection of all images is called the **range** of f. Graphically, the domain of a function is part of the horizontal $x-axis$ whereas the range is part of the $y-axis$. Note that y depends on the value of x. So we call x the **independent** variable and y the **dependent** variable.

Exercise 77
Suppose that $y = -2$ no matter what x is.

(a) Is y a function of x? Explain.
(b) Is x a function of y? Explain.

Exercise 78
Suppose that $x = 2$ no matter what y is.

(a) Is y a function of x? Explain.
(b) Is x a function of y? Explain.

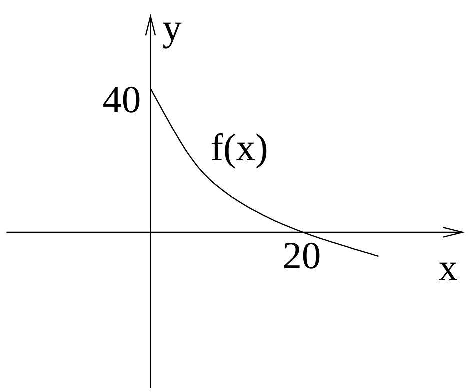
Exercise 79
A function can be described using words. The sales tax on an item is 7.5%. Express the total cost,C, as a function of the price P of the item.

Exercise 80
Suppose you are looking at the graph of y as a function of x.

(a) What is the maximum number of times that the graph can intersect the y axis? Explain.
(b) Can the graph intersect the x axis an infinite number of times? Explain.

Exercise 81
(a) You are going to graph $p = f(w)$. Which variable goes on the horizontal axis and which goes on the vertical axis?
(b) If $10 = f(-4)$, give the coordinates of a point on the graph of f.
(c) If 6 is a solution of the equation $f(w) = 1$, give a point on the graph of f.

Exercise 82
Use the graph to fill in the missing values: (a) $f(0) =?$ (b) $f(?) = 0$.



Exercise 83
It is possible for two quantities to be related and yet for neither quantity to be a

function of the other.
A national park contains foxes that prey on rabbits. The table below gives two populations, F and R, over 12-month period, where $t = 0$ means January 1, $t = 1$ means February 1, and so on.

t (months)	R (rabbits)	F (foxes)
0	1000	150
1	750	143
2	567	125
3	500	100
4	567	75
5	750	57
6	1000	50
7	1250	57
8	1433	75
9	1500	100
10	1433	125
11	1250	143

(a) Is F a function of t? Is R a function of t?
(b) Is F a function of R? Is R a function of F?

Exercise 84
We say that a quantity y is **directly proportional** *to x^n, where n is a positive number, if there exists a constant k such that $y = kx^n$. We call k the* **constant of proportionality.**

(a) Suppose y is directly proportional to x. If $y = 6$ when $x = 4$, find the constant of proportionality and write the formula for y as a function of x.
(b) Suppose that y is directly proportional to the square of d. If $y = 45$ when $d = 3$, find the constant of proportionality and write the formula for y as a function of x.

Exercise 85
We say that a quantity y is **inversely proportional** *to x^n, where n is a positive number, if there exists a constant k such that $y = \frac{k}{x^n}$. We call k the* **constant of proportionality.**

(a) Suppose y is inversely proportional to x. If $y = 6$ when $x = 4$, find the constant of proportionality and write the formula for y as a function of x.
(b) Suppose that y is inversely proportional to the square of d. If $y = 45$ when $d = 3$, find the constant of proportionality and write the formula for y as a function of x.

Exercise 86
For each of the formulas below, state whether y is directly proportional to x and, if so, give the constant of proportionality:

(a) $y = 5x$ *(b)* $y = x \cdot 7$ *(c)* $y = x \cdot x$ *(d)* $y = \sqrt{5}x$ *(e)* $y = \frac{x}{\pi}$ *(f)* $y = \frac{\pi}{x}$ *(g)* $y = x + 2$ *(h)* $y = 3(x + 2)$ *(j)* $y = 6z$ *where* $z = 7x$.

Exercise 87
An astronaut's weight, w, is inversely proportional to the square of his distance, r, from the earth's center. Suppose that he weighs 180 pounds at the earth's surface and that the radius of the earth is approximately 3960 miles.

(a) Find the constant of proportionality k.
(b) If $w = f(r)$, find $f(5000)$.

Exercise 88
The radius, r, of a sphere is directly proportional to the cube root of its volume V.

(a) A spherical tank has radius 10 centimeters and volume 4188.79 cubic centimeters. Find the constant of proportionality and write r as a function of V, that is $r = f(V)$.
(b) The volume of the sphere in part (a) is doubled. What is the new radius?

Exercise 89
Let $f(x) = x^2$. Evaluate $\frac{f(a+h)-f(a)}{h}$. This quantity is known as the **difference quotient** *of f at a or the* **average rate of change** *of f on the interval $[a, a+h]$. This concept is of great importance in calculus.*

Exercise 90
For $f(x) = x^2 - 4x + 7$, find $\frac{f(x+h)-f(x)}{h}$.

Exercise 91
The Fibonacci sequence is a sequence of numbers that begins $1, 1, 2, 3, 5, \cdots$ Notice that each term in the sequence is the sum of the two preceding terms. Let $f(n)$ be the nth term in the sequence. What is the relationship between $f(n), f(n-1)$ and $f(n-2)$?

Exercise 92 *(Defining functions using sums)*
Let $s(n)$ be the sum of the first n positive integers. That is,

$$s(n) = 1 + 2 + 3 + \cdots + n.$$

By writing $s(n)$ in reverse order and adding the resulting expression to the previous expression show that a compact form of $s(n)$ is given by the expression

$$s(n) = \frac{n(n+1)}{2}.$$

Exercise 93 *(Arithmetic Sequences)*
By a **sequence** *we mean a list of numbers: $f(1), f(2), \cdots$ where f is a function. Thus, a sequence is a function with domain the set of nonnegative integers.*

The numbers in the sequence are called **terms**. *Thus,* $f(n)$ *is called the* **nth term.** *Suppose that each term in the sequence is equal to the previous term plus a constant* d. *That is,* $f(n) = f(n-1) + d$. *Such a sequence is called an* **arithmetic sequence.** *Show that* $f(n) = f(1) + (n-1)d$.

Exercise 94
Let $f(1), f(2), \cdots$ *be an arithmetic sequence. Show that*

$$f(1) + f(2) + \cdots + f(n) = \frac{n}{2}[2f(1) + (n-1)d]$$

Exercise 95
If air resistance is neglected, every falling object travels 16 ft during the first second, 48 ft during the next, 80 ft during the next, and so on. These numbers form an arithmetic sequence.

(a) Find d *and* $f(n)$.
(b) Calculate the distance an object falls after three seconds.

Exercise 96 *(Geometric Sequence)*
If a sequence $f(1), f(2), \cdots$ *is such that each term is the previous term times a constant* d, *i.e.* $f(n) = f(n-1)d$, *then we call the sequence a* **geometric sequence.** *Show that for a geometric sequence,* $f(n) = f(1)d^n$.

Exercise 97
Let $f(1), f(2), \cdots$ *be a geometric sequence with ratio* $d\ neq 1$.

(a) Let $S(n) = f(1) + f(2) + \cdots + f(n)$. *Calculate* $dS(n) - S(n)$.
(b) Use part (a) to show that $S(n) = f(1)\frac{1-d^{n+1}}{1-d}$.
(c) Suppose that $-1 < d < 1$. *What happens to* $S(n)$ *in the long run (i.e, when* $n \to \infty$*).*

Exercise 98
The **present value**, $\$P$, *of a future payment,* $\$B$, *is the amount which would have to be deposited (at some interest rate, r) in a bank account today to have exactly* $\$B$ *in the account at the relevant time in the future. If* r *is the interest rate compounded annually and if* n *is the number of years then*

$$B = P(1+r)^n \quad or \quad P = \frac{B}{(1+r)^n}.$$

Suppose that Patrick Ewing's contract with the Knicks guaranteed him and his heirs an annual payment of $3 *million forever. How much would the owners need to deposit in an account today in order to provide these payments?*

Exercise 99 *(Parametric Equations)*
Most of the graphing in this book will be with rectangular equations involving only two variables x *and* y. *Thus, we defined the graph to be the collection of the*

ordered pairs (x, y). Sometimes, the variables x and y are functions of a third variable t. That is, $x = f(t)$ and $y = g(t)$. We call these equations the **parametric equations** *for the curve and the variable t is called the* **parameter.** *Graph the curve with parametric equations $x = \frac{t}{2}$ and $y = t^2 - 3$ with $t \geq 0$. Using these equations, write y in terms of x and use a graphing calculator to graph the curve.*

Exercise 100
Using a graphing calculator, graph the curve $x = \sqrt{t}, y = 2t+3$ where $0 \leq t \leq 3$. Write y in terms of x.

Exercise 101
Find a set of parametric equations of the circle $x^2 + y^2 = 1$.

Exercise 102
When a curve is defined by a formula of the form $y = f(x)$ then we say that it is defined **explicitly.** *When a curve is defined by an expression of the form $f(x, y) = 0$ then we say that the curve is defined* **implicitly.** *Consider the following explicitly defined curve $y = \pm\sqrt{1 - x^2}$ for $-1 \leq x \leq 1$. Find the implicit formula for this curve.*

Exercise 103
Sketch a graph of $|y| = |x|$. Find an explicit formula for y in terms of x.

Exercise 104
Find the domain of the function defined by the set:

$$\{(-3, 0), (-1, 4), (0, 2), (2, 2), (4, -1)\}.$$

Exercise 105
When a function is defined by more then one expression then it is called **piecewise defined function**.*Consider the following piecewise defined function*

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

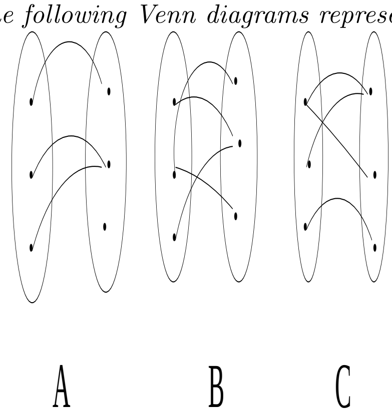
Find $f(0)$.

Exercise 106
Let $f(x) = -x^2 + 4x + 1$. Find $f(x + 2)$.

Exercise 107
Which of the following represents a function?

(A) $x = y^2$ (B) $x^2 + y^2 = 4$ (C) $x + y^2 = 4$ (D) $x = -y^2$

Exercise 108
Determine which of the following Venn diagrams represent a function.



Exercise 109
If $f(x) = 4x^2 - 3$ *then what is* $f(x^2)$*?*

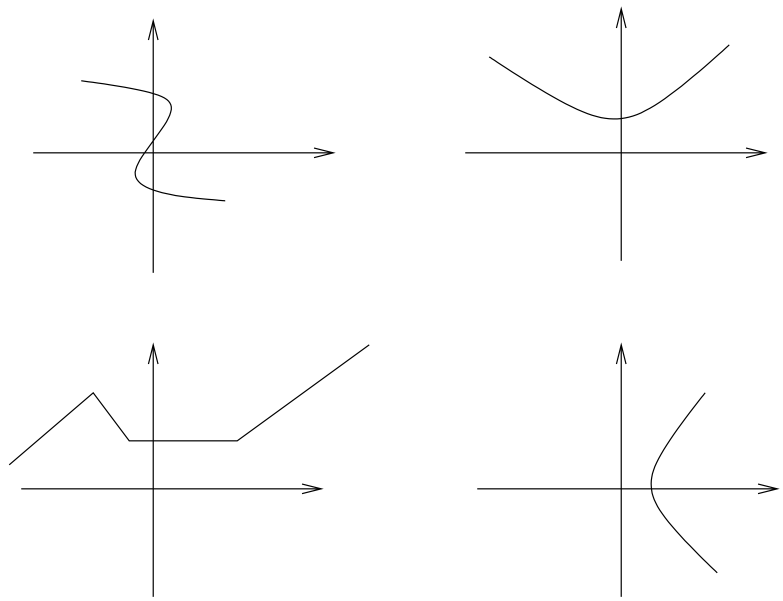
Exercise 110
Find the domain of the function $f(x) = \frac{-1}{(x+3)(x-3)}$.

Exercise 111
Find the domain of the function $f(x) = \frac{\sqrt{4-x}}{(x+1)(x-4)}$.

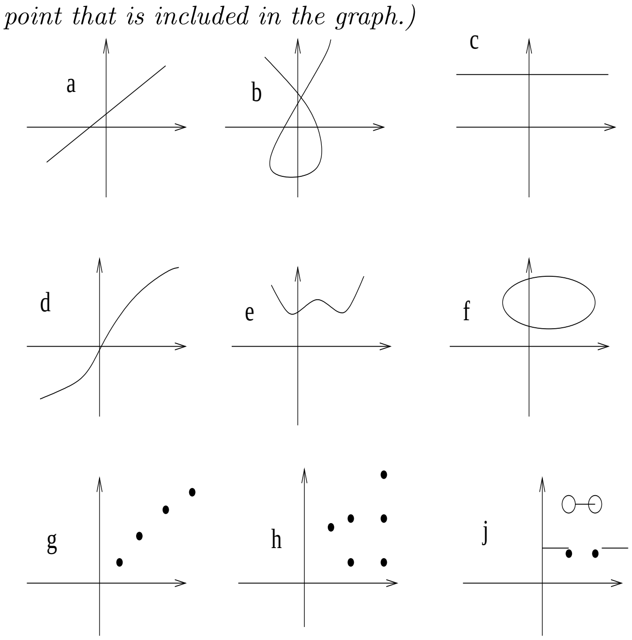
Exercise 112
Find the domain of the function $f(x) = \frac{1}{x}$.

Exercise 113
What is the range of the function $f(x) = |x|$*?*

Exercise 114
To say that y *is a function of* x *means that for each value of* x *must be associated* **exactly** *one value of* y*. What does this requirement mean graphically? In order for a graph to represent a function, each* x *value must correspond to exactly one* $y-$*value. This means that the graph must intersect any vertical line at most once. If a vertical line cuts the graph for example twice, the graph could not be the graph of a function since we have two* y *values for the same* $x-$*value and this violates the definition of a function. The above results in the following test:* **vertical line test***: If there is a vertical line that crosses the graph more than once then the graph is not the graph of a function. Which of the graphs (a) through (d) are graphs of functions?*



Exercise 115
Which of the graphs (a) through (i) represent y as a function of x? (Note that an open circle indicates a point that is not included in the graph; a solid dot indicates a point that is included in the graph.)



Exercise 116
Suppose \$200 *is invested at* 6% *per year simple interest for t years. Find a formula for the earned interest I.*

Exercise 117
A ball thrown vertically upward from the roof of a skyscraper. Let $h(t)$ denote the height of the ball from the ground t seconds after it is thrown. Then, from the laws of physics, $h(t)$ is given by an expression of the form

$$h(t) = -16t^2 + v_0 t + h_0$$

where v_0 is the initial velocity of the ball and h_0 is the initial height of the ball. Suppose that the building is 1200 feet tall. Furthermore, suppose that an observer sees the ball pass by a window 800 feet from the ground 10 seconds after the ball was thrown. Determine a formula for the function $h(t)$.

Exercise 118
Two cars leave an intersection at the same time, proceeding in perpendicular directions. One car is moving at 45 mph, and the other is moving at 30mph. Determine an expression for the function $D(t)$ that gives the distance between the two cars as a function of time.

Exercise 119
For a positive integer n we define n **factorial**, *denoted by $n!$, to be the product $n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. Let $p(n) = n!$. Evaluate $p(n)$ for $1 \leq n \leq 10$. Compile your results in a table. Use a calculator to find* $0!$

Exercise 120
Give an example of a function that cannot be defined by a formula.

Exercise 121
Let $s(t) = 11t^2 + t + 100$ be the position, in miles, of a car driving on a straight road at time t, in hours. The car's velocity at any time t is given by $v(t) = 22t + 1$.

(a) Use function notation to express the car's position after 2 hours. Where is the car then?
(b) Use function notation to express the question, "when is the car going 65 miles per hour?"
(c) Where is the car when it is going 67 mph?

1.6 Chapter Test

Exercise 122
A car is A miles west of Dayton. Going at a constant velocity and proceeding west, the car is B miles west of Dayton after t hours. Express the velocity of the car in terms of A, B and t.

Exercise 123
Write the following expression without using parentheses and brackets: $x - 5[3 - 2(x+2)]$

Exercise 124
Write the following statement as inequality: The length y of the new edition of a book is at most 20 percent longer than the previous edition x.

Exercise 125
Calculate: $|\frac{33}{12} - \pi|$

Exercise 126
Find the distance between the two points $A(-1)$ *and* $B(-5)$.

Exercise 127
Calculate: $|-8| - |17| - |-8 - 17|$.

Exercise 128
Suppose that the coordinates of a point $A(x, y)$ *satisfy* $x > 0, y < 0$. *Locate the quadrant that* A *belong to.*

Exercise 129
Suppose that (x, y) *belongs to the third quadrant. To what quadrant does the point* $(x, -y)$ *belong to?*

Exercise 130
Find the distance between the points $A(6, -3)$ *and* $(6, 5)$.

Exercise 131
Find the midpoint of the line segement joining the points $(-\frac{1}{3}, -\frac{1}{3})$ *and* $(-\frac{1}{6}, -\frac{1}{2})$.

Exercise 132
Find the standard form of the equation of the circle with center $(2, -1)$ *and radius* 4.

Exercise 133
Find the standard form of the equation of the circle with diameter $(-4, -1)$ *and* $(4, 1)$

Exercise 134
Find the center and the radius of the circle: $(x - 1)^2 + (y + 3)^2 = 4$

Exercise 135
A plane flies in a straight line to a city that is 100 kilometers east and 150 kilometers north of the point of departure. How far does it fly?

Exercise 136
Determine the point that lies on the graph of the equation: $y = x^2 - 3x + 2$.

(A) $(2, 0)$ *(B)* $(-2, 8)$ *(C)* $(-1, 6)$ *(D)* $(2, 8)$

Exercise 137
Solve the equation: $2(x + 5) - 7 = 3(x - 2)$.

Exercise 138
Solve the equation: $\frac{x}{x+4} + \frac{4}{x+4} + 2 = 0$

Exercise 139
Solve the equation: $\frac{5x-4}{5x+4} = \frac{2}{3}$.

Exercise 140
Find a common solution to the two equations $y = 2 - x$ *and* $y = 2x - 1$.

Exercise 141
Find x *so that the distance from* $(2, -1)$ *to the point* $(x, 2)$ *is 5.*

Exercise 142
The energy, E*, in foot-pounds delivered by an ocean wave is proportional to the length,* L*, of the wave times the square of its height,* h*.*

(a) Write a formula for E *in terms of* L *and* h*.*
(b) A 30-foot high wave of length 600 feet delivers 4 million foot-pounds of energy. Find the constant of proportionality.

Exercise 143
A meal in a restaurant costs $\$M$*. The tax on the meal is* 5%*. You decide to tip the server* 15%*.*

(a) Write an algebraic expression in terms of M *for the amount you pay the server. Included the cost of the meal, tax, and tip and assume that you tip the waiver* 15% *of the cost of the meal: (i) Not including the tax. (ii) Including the tax.*
(b) In each case in part (a), is the total amount paid proportional to the cost of the meal?

Exercise 144
Suppose $v(t) = t^2 - 2t$ *gives the velocity, in ft/sec, of an object at time* t*, in seconds.*

(a) What is the initial velocity, $v(0)$*?*
(b) When does the object have a velocity of zero?
(c) What is the meaning of the qunatity $v(3)$*? what are its units?*

Chapter 2

Algebraic Expressions

Many of the content of this chapter were covered in previous courses. However, it may be good to review some basic rules and definitions. As you work through the problems, try to make the vocabulary and the manipulations second nature so that you can use them quickly and appropriately.

2.1 Integral Exponents

Exercise 145
Let a be any real number. For any positive integer n we define

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n\ factors}$$

where we call a the **base** *and n the* **exponent***. For $a \neq 0$ we define $a^0 = 1$ and for $n < 0$ we define $a^{-n} = \frac{1}{a^n}$. It is worth noticing that $a^1 = a$ because here we have only one factor of a and $a^{-1} = \frac{1}{a}$.*

(i) Simplify π^0 and $(-\sqrt{3})^0$.
(ii) Write the following with positive exponent: $\frac{1}{(0.75)^{-6}}$ and 3^{-4}.

Exercise 146
Be aware of the following notational conventions:
(i) $-a^n = -(a)^n = (-1)a^n$ and $-a^n \neq (-a)^n$.
(ii) $-ab^n = (-a)(b^n)$.
Thus, the notation -2^4 means 2^4 multiplied by -1. Thus, $-2^4 = -16$. On the other hand, $(-2)^4$ means the product of (-2) by itself four times. That is, $(-2)^4 = (-2)(-2)(-2)(-2) = 16$. Calculate $-3^3 - (-4)^2$.

Exercise 147 *(Rule of Exponentiation)*
The product rule of exponents is represented algebraically by $a^m \cdot a^n = a^{m+n}$. Find the algebraic representation of each of the following rules of exponentiation:

(i) Quotient rule.
(ii) Power rule.
(iii) Raising a product to a power.
(iv) Raising a quotient to a power.

Exercise 148
Is it true that the power of a sum is always equal to the sum of powers? That is, do we always have $(a+b)^n = a^n + b^n$*? If not, give an example. For what value(s) of* n *the equality* $(a+b)^n = a^n + b^n$ *is valid for all* a *and* b*?*

Exercise 149
In order to make the product rule $a^m \cdot a^n = a^{m+n}$ *valid for all integers* m *and* n *and* $a \neq 0$ *then one must define* $a^0 = 1$*. Explain why.*

Exercise 150
Scientific notation *for a number is an expression of the form*

$$N \times 10^n$$

where $1 \leq N < 10$*,* N *is in decimal notation and* n *is positive integer if the number is greater than 10 and a negative integer if the number is between 0 and 1. The mass of a neutron is*

$$0.0000000000000000000000000167$$

Convert this number to scientific notation.

Exercise 151
Convert each of the following to decimal notation:

(a) 7.63×10^{-4}. *(b)* 9.4×10^5.

Exercise 152
Simplify: $(a^{-2})^{-3}$

Exercise 153
Simplify: $(\frac{a^{-3}}{b^5})^{-4}$

Exercise 154
Simplify: $(\frac{a^{-3}c}{b^{-2}})^4$

Exercise 155
Simplify: $(\frac{3a^{-2}}{b^3})^2(\frac{2a^{-1}}{b})^3$

Exercise 156
Write the following expression so that all exponents are positive.

$$\frac{x^5y^{-2}}{x^3y}.$$

Exercise 157
Write the following expression so that all exponents are positive.

$$(\frac{4x}{5y})^{-2}$$

Exercise 158
Write the following expression so that all exponents are positive.

$$\frac{3x^{-2}yz^2}{x^4y^{-3}z}$$

Exercise 159
Write the following expression so that all exponents are positive.

$$(\frac{3x^{-1}}{4y^{-1}})^{-2}$$

Exercise 160
Simplify: $4^{-2}4^3$.

Exercise 161
Simplify: $\frac{2^3\cdot 3^2}{2\cdot 3^{-2}}$.

Exercise 162
Simplify, using no negative exponents: $\frac{(-6x)^0}{(2x^2)^{-1}}$.

Exercise 163
Simplify, using no negative exponents: $(3b^{-3})^2$.

Exercise 164
Simplify, using no negative exponents.

$$(\frac{4a}{b^2})^{-2}(\frac{2b}{a^2})^4$$

Exercise 165
How much must be invested at 10% *compounded semiannually so that* $\$1,000$ *will be accumulated at the end of 2 years? Recall the compound formula* $A = P(1+\frac{r}{n})^{nt}$.

Exercise 166
What is the compound amount on $\$500$ *invested for 2 years at* 12% *compounded quarterly?*

Exercise 167
Suppose that $\$1,000$ *is invested at an annual rate of* 6% *compounded semiannually. How much will have accumulated at the end of 1 year?*

2.2 Radicals and Rational Exponents

Exercise 168
A number c is said to be an **nth root** *of a if* $c^n = a$. *We write* $c = \sqrt[n]{a}$. *We call a the* **radicand** *and the symbol* $\sqrt[n]{}$ *the* **radical**. *Simplify* $\sqrt[5]{\frac{32}{243}}$.

Exercise 169 *(Rules of Radicals)*
Complete the following:

(i) If n is even then $\sqrt[n]{a^n} =$
(ii) If n is odd then $\sqrt[n]{a^n} =$
(iii) $\sqrt[n]{a}\sqrt[n]{b} =$
(iv) $\sqrt[n]{\frac{a}{b}} =$

Exercise 170
Removing the radicals in a denominator or a numerator is called **rationalizing the denominator** *or* **rationalizing the numerator.** *Pairs of expressions of the form* $a + b$ *and* $a - b$ *are called* **conjugates.** *Rationalizing is done by multiplying an expression by its conjugate. Rationalize the denominator:* $\frac{1}{3+\sqrt{2}}$

Exercise 171
Rationalize the numerator: $\sqrt{x+h} - \sqrt{x}$

Exercise 172
Rationalize the denominator: $\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}}$

Exercise 173
The expression $\sqrt[n]{a^m}$ *can be written in terms of a radical exponents such as* $\sqrt[n]{a^m} = a^{\frac{m}{n}}$. *Complete the following:*

(i) $a^{\frac{1}{n}} =$
(ii) $a^{-\frac{m}{n}} =$

Exercise 174
Simplify: $\sqrt[5]{\frac{x^7}{y^8 z^{11}}}$

Exercise 175
Simplify: $(a^4 b^6 c^{\frac{2}{3}})^{\frac{1}{2}}$

Exercise 176
Rewrite using fractional exponent : $\sqrt[5]{(x+y)^2}$

Exercise 177
Rewrite using radicals: $(x^{\frac{1}{2}} + y^{\frac{1}{2}})^{\frac{1}{2}}$

Exercise 178
Find the value of $8^{\frac{2}{3}}$.

Exercise 179
Simplify using no negative exponents in the final answer:

$$\frac{(x^{-\frac{1}{2}}y)^3x^{-\frac{5}{2}}y^4}{(xy^2)^{-2}}$$

Exercise 180
Rationalize the numerator of $\frac{\sqrt{x+h}-\sqrt{x}}{h}$.

Exercise 181
Compute without using a calculator: $(\frac{8}{27})^{\frac{2}{3}}$.

Exercise 182
Compute without using a calculator: $(9)^{-\frac{1}{2}}$.

Exercise 183
Compute without using a calculator: $(-1000)^{-\frac{1}{3}}$

Exercise 184
Simplify $\sqrt[3]{-16}$.

Exercise 185
Simplify :$\sqrt[3]{-\frac{8}{27}}$

Exercise 186
Rationalize the numerator: $\frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$.

2.3 Polynomials

Exercise 187
A **polynomial** *in the variable x of* **degree** *n is any expression of the form*

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0, \quad a_n \neq 0.$$

We call a_n the **leading coefficints**. *Find the leading term and the degree of the polynomial* $\pi x^5 - 3x^2 - \frac{1}{2}$.

Exercise 188
If two terms in an expression have the same variables raised to the same powers then they are called **like terms.** *We add and subtract polynomials by combining like terms.*
Simplify: $(x^2 + 2x - 5) - (x^2 + x - 1)$

Exercise 189
Simplify: $(-5a^2b^3)(-7ab^2)$

Exercise 190
Multiplication of polynomials is based on the distributive property. Simplify: $(3x - 1)(x^2 + x - 2)$

Exercise 191
Expand: $(x + \sqrt{5})^2$.

Exercise 192
Simplify: $(x + 3)(x - 3)$

Exercise 193
Multiply: $(x^2 + 3y)(x^2 - 3y)$.

Exercise 194
To factor a ploynomial is to write it as a product of factors. Important identities used in factoring are:

(i) $x^2 - a^2 = (x - a)(x + a)$
(ii) $(x + a)^2 = x^2 + 2ax + a^2$
(iii) $(x - a)^2 = x^2 - 2ax + a^2$
(iv) $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$
(v) $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$.

Factor: $4x^2 - 9$

Exercise 195
Factor $-a - b$.

Exercise 196
Another way to factor algebraic expressions is to use the idea of grouping terms. Factor $x^2 - hx - x + h$.

Exercise 197
Factor $hx^2 + 12 - 4hx - 3x$.

Exercise 198
One way to factor quadratics is to mentally multiply out the possibilities. For example, $x^2 + ax + b = (x - c)(x - d)$ *where* $cd = b$ *and* $c + d = a$.
Factor: $4x^2 - 12x + 9$

Exercise 199
Factor: $x - 3$, *where* $x \geq 0$.

Exercise 200
An ordered triple (a, b, c) *of positive integers is called a* **Pythagorean triple** *if* $a^2 + b^2 = c^2$. *A Pythagorean triple can be used as lengths of a right triangle. Determine which of the following triple is a Pythagorean triple.*

(A) $(1, 2, 3)$ *(B)* $(2, 3, 4)$ *(C)* $(5, 6, 7)$ *(D)* $(3, 4, 5)$ *(E) None of the above.*

Exercise 201
In a right triangle one leg is of length 4 and the other is of length 3. What is the length of the hypotenuse?

Exercise 202
Find the length of the hypotenuse in a right triangle whose sides are of lengths 5, 12, and 13.

Exercise 203
Let a, b, c be the lengths of the sides of a right triangle with c being the length of the hypotenuse. Find a given that $b = 7$ and $c = 25$.

Exercise 204
Find the diagonal of a rectangle whose length is 8 inches and whose width is 5 inches.

Exercise 205
Which of the following is a polynomial?

(A) $x^2 - \sqrt{x} + 5$ *(B)* $x^3 - \frac{1}{x} + 1$ *(C)* $x^2 - 4x + 10$ *(D)* $x^2 + \sin x$ *(E) None of the above.*

Exercise 206
Simplify: $(3x^4 - 4x^3 + 6x^2 - 1) - (2x^4 - 8x^2 - 6x + 5)$

Exercise 207
Find the product: $(2x + 5)(x^2 - x + 2)$

Exercise 208
Factor: $x^4 - 16$

2.4 More of Factoring

Exercise 209
Factor $4x^2 - 7$

Exercise 210
Factor $x^4 + 2x^2y^2 + y^4$

Exercise 211
Factor $a^{2n} - 2a^n + 1$

Exercise 212
Factor $a^3 - 8b^3$

Exercise 213
Factor $x^4 + 8x$

Exercise 214
Factor $a^2 - 2ab + b^2 - 1$

Exercise 215
Factor $x^2 - 6x + 9$

Exercise 216
Factor $x^2 + x - 12$

Exercise 217
Factor $x^3 - 9x^2 + 27x - 27$

Exercise 218
Suppose that $f(x) = x^3$. *Calculate* $\frac{f(x+h)-f(x)}{h}$.

Exercise 219
Factor: $27x^9 - 8y^{12}$

Exercise 220
Factor: $x^4 - 4y^4$

Exercise 221
Factor: $x^3y - 4xy^3$

Exercise 222
Factor: $2x^3 - x^2 - 8x + 4$

Exercise 223
Factor: $x^2(x^2 - 9) - 16(x^2 - 9)$

Exercise 224
Factor: $4x^4 + 12x^2y^2 + 9y^4$

2.5 Rational Expressions

Exercise 225
A **rational expression** *is the ratio of two ploynomials. The domain of a rational expression consists of all numbers that make the denominator nonzero. Find the domain of the expression* $\frac{x^2-4}{x^2-4x-5}$.

Exercise 226
Simplify: $\frac{4-2x}{x-2}$.

Exercise 227
Simplify: $\frac{x^2-4}{x-2}$.

Exercise 228 *(Splitting Fractions)*
To split a fraction of the form $\frac{a+b}{c}$ *means to write it in the form*

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}.$$

Split the fraction $\frac{3x^2+2}{x^3}$ *into two reduced fractions.*

Exercise 229
Simplify: $\frac{x^2}{x+1} \cdot \frac{x^2+2x+1}{x}$.

Exercise 230
Simplify: $\frac{\frac{x}{x+2}}{\frac{x^2}{x^2-4}}$.

Exercise 231
Simplify: $\frac{x-1}{x} + \frac{1}{x}$.

Exercise 232
Perform the indicated operation and simplify:

$$\frac{x}{x^2-y^2} - \frac{y}{x^2+2xy+y^2}$$

Exercise 233
Simplify:

$$\frac{\frac{5}{2}-\frac{2}{3}}{\frac{3}{4}+\frac{1}{6}}.$$

Exercise 234
Simplify: $\frac{(x+1)^3}{x^3+1}$.

Exercise 235
Simplify:

$$\frac{\frac{2}{c}-\frac{2}{d}}{c-d}$$

Exercise 236
Simplify:

$$\frac{x+y}{\frac{1}{x^2}-\frac{1}{y^2}}$$

Exercise 237
Simplify: $(\frac{a^{n^2-1}}{a^{1-n}})^{\frac{1}{n-1}}$

Exercise 238
Simplify:

$$\frac{x \cdot \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x - (x^2+1)^{\frac{1}{2}}}{x^2}$$

Exercise 239
Simplify: $(\frac{c^{n^2-2}}{c^{3n-4}})^{\frac{1}{n-2}}$

Exercise 240
Simplify:

$$\frac{x^{4n}y^n - x^n y^{4n}}{x^n y^{3n} - x^{3n} y^n}$$

Exercise 241
Let $f(x) = \frac{x}{x+1}$. *Find and simplify* $\frac{f(x+h)-f(x)}{h}$.

2.6 Chapter Test

Exercise 242
Juanita's weekly salary increases from \$205 *to* \$213.20. *What is the percent of increase?*

Exercise 243
Simplify: $(3x^3 - 5x^2 + 8x - 3) - (5x^3 - 7x + 11)$

Exercise 244
Find the product: $(x^4 - 5x^2 + 7)(3x^2 + 2)$

Exercise 245
Factor: $u(v + w) + 7v(v + w)$

Exercise 246
Factor:$16x^4 - (3y + 2z)^2$

Exercise 247
Factor: $2x^2 + 9x + 4$

Exercise 248
Factor: $6x^2 + 13x - 5$

Exercise 249
Factor: $4x^3 - 8x^2 - x + 2$

Exercise 250
Factor: $x^4 + 6x^2y^2 + 25y^4$

Exercise 251
Reduce the following fraction to lowest terms: $\frac{5x^2-14x-3}{2x^2+x-21}$

Exercise 252
Perform the indicated operation and simplify the result:

$$\frac{x^2 - 49}{x^2 - 5x - 14} \div \frac{2x^2 + 15x + 7}{2x^2 - 13x - 7}$$

Exercise 253
Perform the following operation and simplify:

$$\frac{3x}{4x - 1} + \frac{2x}{3x - 5}$$

Exercise 254
Perform the following operation and simplify:

$$\frac{x}{x^3 + x^2 + x + 1} - \frac{1}{x^3 + 2x^2 + x} - \frac{1}{x^2 + 2x + 1}$$

Exercise 255
Simplify: $\frac{1+\frac{1}{x}}{x-\frac{1}{x}}$

Exercise 256
Simplify:

$$\frac{\frac{1}{x^3}+\frac{2}{x^2y}+\frac{1}{xy^2}}{\frac{y}{x^2}-\frac{1}{y}}$$

Exercise 257
Simplify: $\sqrt[7]{\sqrt[5]{x^{35}}}$

Exercise 258
Simplify: $\sqrt[5]{a^4}\sqrt[5]{a^3}$

Exercise 259
Simplify: $7\sqrt{12}+\sqrt{75}-5\sqrt{27}$

Exercise 260
Expand and then simplify: $(3\sqrt{x}-7\sqrt{y})(5\sqrt{x}+2\sqrt{y})$

Exercise 261
Rationalize the denominator:$\frac{2}{\sqrt[3]{x}+2}$

2.7 Cumulative Test

Exercise 262
Factor the following expression and simplify the result:

$$(y+2)^{-\frac{2}{3}}(y+1)^{\frac{2}{3}}+2(y+2)^{\frac{1}{3}}(y+1)^{-\frac{1}{3}}$$

Exercise 263
Rewrite the following expression without using parentheses:

$$3[-(2b+c)+(c-b)-b]$$

Exercise 264
Simplify so that all exponents are positive.

$$(\frac{a^3}{b^6c^4})^{-\frac{2}{3}}(\frac{8a^{\frac{1}{3}}b^{\frac{3}{2}}}{c^{\frac{1}{3}}})^6$$

Exercise 265
Express the following number as the ratio of two positive integers: $0.\overline{123}$

Exercise 266
Find the value of the following expression without using a calculator: $(\frac{8}{27})^{\frac{2}{3}}$

Exercise 267
Simplify: $\frac{5}{2}(-2\frac{1}{3})$

Exercise 268
Simplify so that all exponents are positive.

$$(\frac{x^{-2}}{y^3})^{-2}(\frac{x^{-3}}{y^{-4}})^{-3}$$

Exercise 269
Simplify: $1\frac{3}{8} - 2\frac{5}{6} - 1$

Exercise 270
Simplify so that all exponents are positive.

$$\frac{(5x^2)^{-2}(5x^5)^{-2}}{(5^{-1}x^{-2})^2}$$

Exercise 271
Write a formula for the following statement: The area A *of a circle of radius* r

Exercise 272
Rationalize the numerator: $\frac{3\sqrt{x}+\sqrt{y}}{5}$

Exercise 273
Rewrite in terms of exponential: $(2a+1)(2a+1)(2a+1)$

Exercise 274
Perform the indicated operations and simplify the result: $(\sqrt{a+b} - \sqrt{a})^2$

Exercise 275
Simplify: $\frac{xy^{-2}}{x^{-3}y}$

Exercise 276
Simplify: $\sqrt[3]{\frac{(a+b)^9}{27a^3}}$

Exercise 277
How much must be invested at 6% *compounded monthly so that* \$500 *accumulates at the end of 6 months?*

Exercise 278
Factor completely: $9x^2y^2 - 12xy^4$

Exercise 279
Simplify:$(4\frac{1}{3} + 1\frac{2}{5}) \div (2\frac{1}{2})$

Exercise 280
Expand: $(xy^2 + 3)(2xy^2 + 1)$

Exercise 281
Use the properties of exponents to simplify: $(-x^2y)(x^4y^3)$

Exercise 282
New York state income tax is based on what is called taxable income. A person's taxable income is part of his/her total income; the tax owed to the state is calculated using the taxable income (not total income). For a person with a taxable income between \$65,000 *and* \$100,000*, the tax owed is* \$4,635 *plus* 7.875% *of the taxable income over* \$65,000.

(a) Compute the tax owed by a lawyer whose taxable income is \$68,000.
(b) Consider a lawyer whose taxable income is 80% *of her total income,* x*, where* x *is between* \$85,000 *and* \$120,000*. Write a formula for* $T(x)$ *the taxable income.*
(c) Write a formula for $L(x)$*, the amount the owed by the lawyer in part (a).*
(d) Use $L(x)$ *to evaluate the tax liability for* $x = 85,000$ *and compare your results to part (a).*

Chapter 3

Equations and Inequalities

In this section we discuss the skills required to solve both equations and inequalities.

3.1 Quadratic Equations

Exercise 283
A **quadratic equation** *is a second degree polynomial expression of the form*

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$. Give an example of a quadratic equation with leading coefficient -3.

Exercise 284
If a number α satisfies a quadratic equation then we call it a **solution, a zero,** *or a* **root.** *Show that the numbers $-\sqrt{k}$ and $\sqrt{k}$ satisfy the equation $x^2 = k,\ k \geq 0$.*

Exercise 285
Solve $3x^2 = 27$

Exercise 286
To solve a quadratic equation means to find its solutions. One way for doing that is by using the method of completing the square. The method requires that the leading coefficient is 1. Then the terms with x are set on the left hand side of the equation. Add to both sides the square of half the coefficient of x to obtain a complete square of the form $(x + \alpha)^2 = k$. If $k \geq 0$ then the solutions are $x = -\alpha \pm \sqrt{k}$. Otherwise, the equation has no solutions. Solve by the method of completing the square:$x^2 = 12 - x$

Exercise 287
Solve by completing the square: $x^2 - 6x + 9 = 0$

Exercise 288
Solve by completing the square: $3x^2 - 5x + 1 = 0$

Exercise 289
Solve by completing the square: $3x^2 + 2 = 4x$

Exercise 290
Another way for solving quadratic equations is by factoring. In this process we assume the equation is of the form $x^2 + dx + e = 0$. *We look for two numbers* α *and* β *such that* $\alpha + \beta = d$ *and* $\alpha \cdot \beta = e$. *Thus,* $x^2 + dx + e = (x - \alpha)(x - \beta) = 0$. *By the Zero Product property* $x = \alpha$ *or* $x = \beta$. *Solve by factoring:* $x^2 + 5x + 6 = 0$

Exercise 291
Solve by factoring: $x^2 + 2x - 15 = 0$

Exercise 292
Solve by factoring: $x^2 - 4x + 3 = 0$

Exercise 293
Solve by factoring: $4x^2 - x - 3 = 0$

Exercise 294
Solve by completing the square: $x^2 + 3x + 2 = 0$

Exercise 295
Solve by completing the square: $x^2 - 6x - 5 = 0$

Exercise 296
The quantity $\Delta = b^2 - 4ac$ *is called the* **discriminant**. *If* $\Delta < 0$ *then the quadratic equation has no solutions. If* $\Delta = 0$ *then the quadratic equation has one single solution given by* $x = -\frac{b}{2a}$. *If* $\Delta > 0$ *then the equation has two different solutions given by the* **quadratic formula** $x = \frac{-b \pm \sqrt{\Delta}}{2a}$. *Solve using quadratic formula:* $x^2 + 3x + 1 = 0$

Exercise 297
Solve using quadratic formula: $2x^2 + 3x - 2 = 0$

Exercise 298
Solve using quadratic formula: $x^2 - 2\pi x + \pi^2 = 0$

Exercise 299
Solve using quadratic formula: $x^2 + \sqrt{2}x + \pi = 0$

3.2 Miscellaneous Equations

Exercise 300
A **radical equation** *is an equation in which variables appear in one or more*

radicals. This type of equations is usually solved by using the so called **Principle of Powers:** *If $a = b$ then $a^n = b^n$ where n is a positive integer. We must check for extraneous solutions at the end.*
Solve: $\sqrt{2x+3} = 7$.

Exercise 301
Solve: $2\sqrt{x} = x - 3$.

Exercise 302
Solve: $\sqrt{x+4} = x - 2$.

Exercise 303
Solve: $\sqrt{3x-3} - \sqrt{x} = 1$.

Exercise 304
Solve: $\sqrt{2x} = 2 - \sqrt{x-2}$

Exercise 305
a **rational equation** *is an equation that contains rational expressions. This type of equations is solved by multiplying through by the Least Common Denominator (also known as the least common multiple). However, when solving this type of equations one must check for extraneous solutions at the end.*
Solve: $\frac{x(x-2)}{2} + \frac{x^2-4}{3} = \frac{x^2+x-6}{5}$

Exercise 306
Solve: $\frac{2x}{x+1} - 3 = \frac{2}{x^2+x}$.

Exercise 307
Solve the equation: $\frac{x^2}{x-3} - \frac{9}{x-3} = 0$.

Exercise 308
Solve: $\frac{x-1}{3x-2} + \frac{3x^2-3}{3x-2} = \frac{3x-3}{3x-2}$

Exercise 309
Solve: $\frac{x}{x-5} = \frac{3x-10}{x-5}$

Exercise 310
Solve: $\frac{1}{x+2} = \frac{4}{x} + \frac{1}{2}$

Exercise 311
Solve: $\frac{1}{x} + \frac{1}{x+1} = \frac{-1}{x(x+1)}$

Exercise 312
Some equations can be treated as quadratic, provided that we make a suitable substitution.
Solve: $y^4 - 3y^2 - 4 = 0$.

Exercise 313
Solve: $x^{-\frac{2}{3}} + 3x^{-\frac{1}{3}} - 10 = 0$.

Exercise 314
Solve: $(1+\frac{1}{y})^2 + 5(1+\frac{1}{y}) + 6 = 0$

Exercise 315
Absolute value equations are equations that involve the absolute value function. If $u(x)$ *is an expression in* x *and* $a \geq 0$ *then the equation* $|u(x)| = a$ *is equivalent to* $u(x) = a$ *or* $u(x) = -a$. *If* $a < 0$ *then the equation has no solutions.*
Solve: $|x-2| = 5$

Exercise 316
Solve : $|x+4| = 13$

Exercise 317
Solve: $|2x-5| = 17$

Exercise 318
Solve: $|2x+7| = |1-x|$

Exercise 319
Solve: $|x-5| = -4$

Exercise 320
How much water must be evaporated from 32 ounces of a 4% *salt solution to make a* 6% *salt solution?*

Exercise 321
How much water should be added to 1 gallon of pure antifreeze to obtain a solution that is 60% *antifreeze?*

3.3 Linear and Absolute Value Inequalities

Exercise 322
Sets of real numbers can be expressed as **intervals.** *For example, if* a *and* b *are real numbers such that* $a < b$ *then the* **open interval** (a, b) *consists of all real numbers between, but not including,* a *and* b. *In set-builder notation we write*

$$(a, b) = \{x | a < x < b\}.$$

The points a *and* b *are the* **endpoints** *of the interval. The parentheses indicate that the endpoints are not included in the interval. If an endpoint is to be included then we use a bracket instead of a parenthesis. Find the set-builder notation of the following intervals and then graph on the real line:*

(a) $(a, b]$ *(b)* $[a, b)$ *(c)* $[a, b]$

Exercise 323
Some intervals extend without bound in one or both directions. For example, the

interval $[a, \infty)$ *begins at* a *and extends to the right without bound. In set-builder notation we have*

$$[a, \infty) = \{x|x \geq a\}.$$

Write the set-builder notation of the following intervals:

(i) (a, ∞) *(ii)* $(-\infty, b)$ *(iii)* $(-\infty, b]$ *(iv)* $(-\infty, \infty)$

Exercise 324
Write interval notation for each set:

(i) $\{x| - 10 \leq x < 5\}$.
(ii) $\{x|x > -2\}$.
(iii) $\{x|x \neq 7\}$.

Exercise 325
A linear inequality is a linear equation with the equal sign replaced by an inequality symbol such as $<, >, \leq,$ *or* $\geq$ *. For example,* $ax + by \leq c$*. Two important principles for solving inequalities:*

Addition Principle: If $a < b$ *then* $a + c < b + c$ *and* $a - c < b - c$*.*
Multiplication Principle: If $a < b$ *and* $c > 0$ *then* $ac < bc$*. If* $c < 0$ *then* $ca > cb$*. Thus when multiplying by a negative number, we must reverse the inequality sign.*
Solve the inequality: $-5x + 6 > 10$*. Write the answer in interval notation.*

Exercise 326
Solve the inequality: $\frac{7}{3}x + 5 \leq \frac{1}{2}(3x - 7)$*. Write the answer in interval notation.*

Exercise 327
Solve the inequality: $\frac{x}{2} + \frac{2x-1}{5} \geq \frac{x}{10}$

Exercise 328
Solve the inequality: $x(x - 1) \geq x^2 + 2x - 5$

Exercise 329
Solve the inequality: $6 - x \leq 2x \leq 9 - x$

Exercise 330
Solve the inequality: $\frac{3x-2}{5} + 3 \geq \frac{4x-1}{3}$

Exercise 331
Solve the inequality: $4x + 7 \geq 2x - 3$.

Exercise 332
Solve the inequality: $-5 < 3x - 2 < 1$

Exercise 333
Sometimes inequalities contain absolute value notation. In this case the following properties are used to solve them:

- $|u| < a$ *is equivalent to* $-a < u < a$.
- $|u| > a$ *is equivalent to* $u < -a$ *or* $u > a$. *Here* $a > 0$.

Similar statements hold if $<$ *and* $>$ *are replaced by* $\leq$ *and* $\geq$.
Solve the inequality: $|x| \leq 5$

Exercise 334
Solve the inequality: $|x - 2| \leq 5$

Exercise 335
Solve the inequality: $|x + 4| > 3$

Exercise 336
Commonwealth Edison Company's energy charge for electricity is 10.494 *cents per kilowatt-hour. In addition, each monthly bill contains a customer charge of* \$9.36. *If your bill ranged from a low of* \$80.24 *to a high of* \$271.80, *over what range did usage vary (in kilowatt-hour)?*

Exercise 337
In your Economics 101 class, you have scores of $68, 82, 87,$ *and* 89 *on the first four of five tests. To get a grade of B, the average of the first five test scores must be greater than or equal to 80 and less than 90. Solve an inequality to find the range of the score you need on the last test to get a B.*

Exercise 338
The percentage method of withholding for federal income tax (1998) states that a single person whose weekly wages, after subtracting withholding allowances, are over \$517, *but not over* \$1,105, *shall have* \$69.90 *plus* 28% *of the excess over* \$517 *withheld. Over what range does the amount withheld vary if the weekly wages vary from* \$525 *to* \$600 *inclusive?*

3.4 Polynomial Inequalities

A ploynomial inequality is a polynomial equation with the equality sign replaced by one the inequality symbols. To solve a polynomial inequality one first finds the zeros of the polynomial equation. These zeros divide the x-axis into intervals in which the sign of the polynomial is tested using test values. Finally, determine the intervals for which the inequality is satisfied and write the solution set in interval notation. Remember to use brackets when endpoints are in the solution set(this usually occurs with inequalities such as $\leq$ or $\geq$).

Exercise 339
Solve $(x - 2)x \leq (x - 2)x^2$.

Exercise 340
Solve : $x(x-1) > 6$.

Exercise 341
Solve: $x^3 + 25x \leq 10x^2$.

Exercise 342
Solve: $x^2 + x - 12 > 0$.

Exercise 343
Solve: $x^2 \leq 4x + 12$

Exercise 344
Rational inequalities *are inequalities that involve rational expressions. To solve a rational inequality we start by simplifying the inequality so that we get 0 on the right side and a fraction on the left side. Next find the values of x that make the numerator zero or the denominator undefined. Now proceed with these values in a similar fashion as the proecdure of solving a polynomial inequality. Be careful not to use brackets at endpoints where the rational expression is undefined.*

Solve: $\frac{x}{x+3} + 2 \leq 0$.

Exercise 345
Solve: $\frac{x}{x-1} \geq \frac{5}{x+5}$.

Exercise 346
Solve: $\frac{(x+3)(2-x)}{(x-1)^2} > 0$.

Exercise 347
Solve: $\frac{4x+5}{x+2} \geq 3$.

3.5 Chapter Test

Exercise 348
The area between two concentric circles equals twice the area of the smaller circle. The radius of the smaller circle is r. What is the radius of the larger circle?

Exercise 349
For which values of k does $2x^2 + kx + 1 = 0$ have exactly one real solution?

Exercise 350
Solve $S = 2\pi rh + 2\pi r^2$ *for* r

Exercise 351
Solve: $x^2 = 5x$

Exercise 352
Solve: $x^2 - 2x = 2.$

Exercise 353
Express the surface area $(S = 6x^2)$ *of a cube in terms of the volume* $(V = x^3)$

Exercise 354
If a right circular cone has height h *and base of radius* r*, then the lateral surface area is given by* $S = \pi r\sqrt{r^2 + h^2}$*. Solve this equation for* h*.*

Exercise 355
Solve: $|4 - x| = -1.$

Exercise 356
Solve $|x + 1| = |x - 2|$

Exercise 357
Solve: $(x - 1)^{\frac{2}{3}} = 2.$

Exercise 358
Find all values of k *for which* $-x^2 + 5x + k = 0$ *has no real solution.*

Exercise 359
Solve: $|x - 5| < |x + 1|.$

Exercise 360
Solve: $2 < |3x - 1| < 5.$

Exercise 361
Find the domain: $\sqrt{3x + 4} + \frac{1}{\sqrt{2-x}}$

Exercise 362
Solve: $\frac{1}{3}(x + 1) \geq \frac{1}{4}(x - 1)$

Exercise 363
Solve: $|x| > \frac{1}{x}$

Exercise 364
Find all values of k *for which* $x^2 + kx + 1 = 0$ *has two real solutions.*

Exercise 365
Find the domain: $\sqrt{x^2 - x - 12}$

Exercise 366
Solve: $\frac{x^2-4}{1-|x|} \geq 0.$

Exercise 367
Find the domain: $\sqrt{\frac{x^2-4}{1-|x|}}$

3.6 Cumulative Test

Exercise 368
Simplify: $(\frac{x}{y^2})^{-3n}(\frac{y^3}{x})^{-n}$

Exercise 369
In $3\frac{1}{2}$ *years you will need* $\$3,000$ *and you want to take care of this by investing now in an account that pays* 10% *interest compounded monthly. How much should you invest?*

Exercise 370
Rewrite the following percent as a decimal: 3.15%

Exercise 371
Rationalize the numerator: $\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}$

Exercise 372
Perform the indicated operations and simplify:

$$\frac{(x+1)^2}{\frac{x\sqrt{x}+1}{2x\sqrt{x}} - \frac{\sqrt{x}-1}{2\sqrt{x}}}$$

Exercise 373
Simplify: $\sqrt{\sqrt[4]{x^{24}}\sqrt[n]{x^{4n}}}$

Exercise 374
Reduce the fraction to lowest terms: $\frac{x^2+5x+6}{x^2+4x+4}$

Exercise 375
Factor: $36x^2 - y^4$

Exercise 376
Expand: $(2x+3)(x-1)(x-2)$

Exercise 377
Simplify so that all exponents are positive: $(a^{-\frac{1}{2}} - b^{-\frac{1}{2}})(a^{-\frac{1}{2}} + b^{-\frac{1}{2}})$

Exercise 378
Factor: $a^3b + 2a^2b + a^2b^2$

Exercise 379
Simplify: $\frac{\frac{1}{x+h)^2} - \frac{1}{x^2}}{h}$

Exercise 380
What can we conclude about the radius of a circle if we know that the area exceeds that of a square whose edges are each 5 centimeters?

Exercise 381
Find the domain: $\sqrt[4]{\frac{3x+1}{1-x}}$

Exercise 382
Find all values of k *for which* $kx^2 - 3x + 2 = 0$ *has two real solutions.*

Exercise 383
Find the domain: $\frac{3}{\sqrt{x}} - \frac{4}{\sqrt{3x-6}}$

Exercise 384
Express the volume ($V = \frac{4}{3}\pi r^3$) *of a sphere in terms of the surface area* ($S = 4\pi r^2$) *of the sphere.*

Exercise 385
Solve: $|3x + 1| = -x$

Exercise 386
Solve: $(1 + \frac{1}{x})^2 + 5(1 + \frac{1}{x}) + 4 = 0$

Exercise 387
Write a quadratic equation with the solutions 2 *and* -7

Chapter 4

Graphs of Functions

When one quantity depends on another quantity in such a way that for a given value of one of them leads to a unique value of the other then we say that one quantity is a function of the other quantity. A function can be represented in several ways: in words, by a graph, by a formula, or by a table of numbers. In this chapter, we discuss two families of functions: linear functions and quadratic functions. In the last section we discuss how functions are combined to create new functions.

4.1 Lines

Exercise 388
A function of the form $f(x) = mx + b$ is called a **linear function.** *The graph of a linear function is a straight line. So equal increments in x corresponds to equal increments in $y = f(x)$.*
Which of the following tables could represent a linear function?

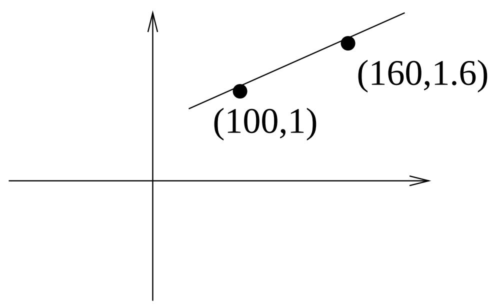
x	*f(x)*
0	*10*
5	*20*
10	*30*
15	*40*

x	*g(x)*
0	*20*
10	*40*
20	*50*
30	*55*

Exercise 389
The **slope** *of a line is a measure to its steepness. If two points (x_1, y_1) and (x_2, y_2) are on the line then the slope is the quantity $\frac{y_2-y_1}{x_2-x_1}$ provided that the*

line is nonvertical. The slope of a vertical line is undefined.
Find the slope of the line through $(1, 2)$ *and* $(5, -3)$.

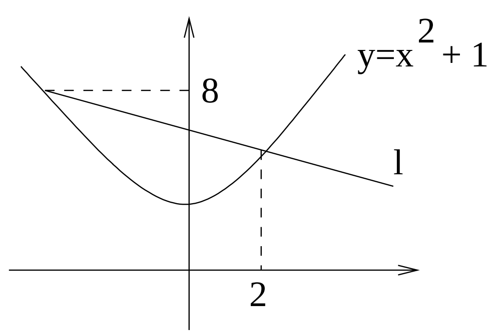
Exercise 390
We can calculate the slope,m*, of a linear function by using the coordinates of two points on its graph. Having found* m *we can use either of the points to calculate* b*, the vertical intercept. Find the formula of* $f(x)$ *given its graph*



Exercise 391
If the slope of a line is m *and the line passes through the point* (x_1, y_1) *then the equation of the line is given by* **point-slope form** *of the line:* $y - y_1 = m(x - x_1)$. *Find an equation of the line through the point* $(1, 2)$ *and with slope 4.*

Exercise 392
Find an equation of the line through the points $(-1, 1)$ *and* $(2, 13)$. *Write the answer in* **standard form** $Ax + By + C = 0$.

Exercise 393
Find the equation of the line l



Exercise 394
Find the equation of the horizontal line passing through the point $(3, 2)$.

Exercise 395
Find the slope m *and the y-intercept* b *of the line* $2x + 4y - 8 = 0$

Exercise 396
Find the equation of the line with slope -2 *and y-intercept 4.*

Exercise 397
Non vertical lines are **parallel** *if and only if they have the same slope.*
Find an equation of the line that contains the point $(2, -3)$ *and is parallel to the line* $2x + y - 6 = 0$.

Exercise 398
The line $y = 3x - 5$ *is parallel to the line:*

(A) $y = -3x - 5$ *(B)* $y = -3x + 5$ *(C)* $x - \frac{1}{3}y - 1 = 0$ *(D)* $y = -3x$

Exercise 399
Find an equation of the line that contains the point $(-1, 3)$ *and is parallel to the line* $2x + y - 1 = 0$.

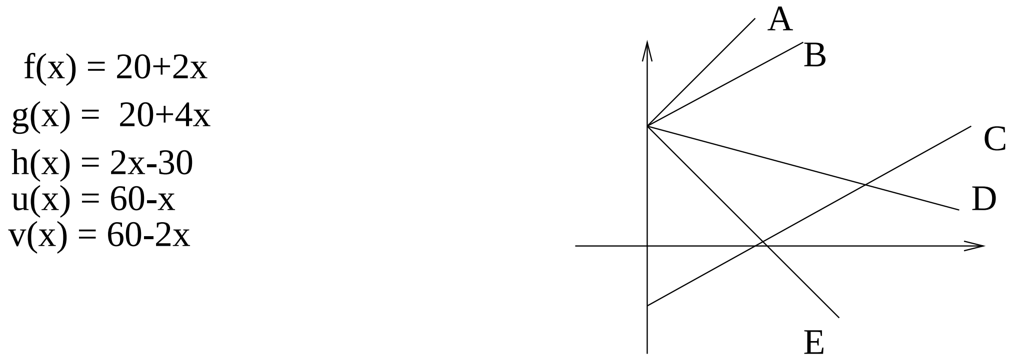
Exercise 400
Two lines are **perpendicular** *if and only if the product of their slopes is -1.*
Find an equation of the line that contains the point $(1, -2)$ *and is perpendicular to the line* $x + 3y - 6 = 0$.

Exercise 401
To find the point at which two lines intersect, notice that the (x, y)*-coordinates of such a point must satisfy the equations for both lines. Thus, in order to find the point of intersection algebraically, solve the equations simultaneously.*
Find the point of intersection of the lines $y = 3 - \frac{2}{3}x$ *and* $y = -4 + \frac{3}{2}x$.

Exercise 402
Two lines are given by $y = b_1 + m_1x$ *and* $y = b_2 + m_2x$, *where* b_1, b_2, m_1, m_2 *are constants.*

(a) What conditions are imposed on b_1, b_2, m_1, m_2 *if the two lines have no points in common?*
(b) What conditions are imposed on b_1, b_2, m_1, m_2 *if the two lines have all points in common?*
(c) What conditions are imposed on b_1, b_2, m_1, m_2 *if the two lines have exactly one point in common?*
(d) What conditions are imposed on b_1, b_2, m_1, m_2 *if the two lines have exactly two points in common?*

Exercise 403
The figure below gives five different lines $A, B, C, D,$ *and* E. *Match each line to one of the following functions* $f, g, h, u,$ *and* v :



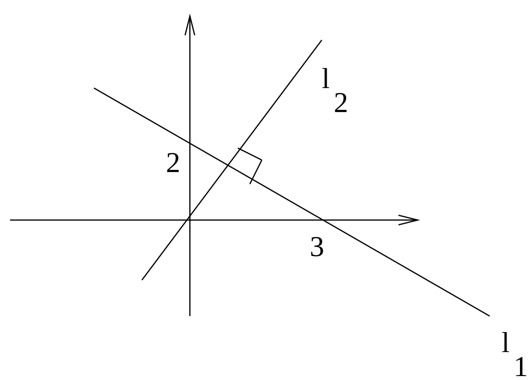
Exercise 404
Find the equation of the line containing the centers of the two circles

$$x^2 + y^2 - 4x + 6y + 4 = 0$$

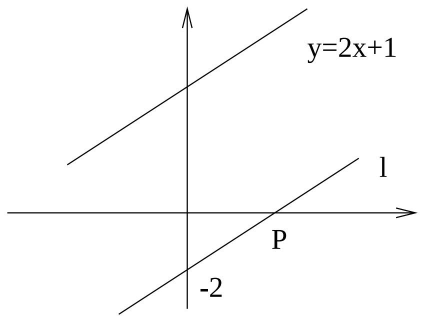
and

$$x^2 + y^2 + 6x + 4y + 9 = 0.$$

Exercise 405
Find the equation of the line l_2 in the figure below.



Exercise 406
Line l in the figure below is parallel to the line $y = 2x + 1$. Find the coordinates of the point P.



Exercise 407
Find the linear function $f(x) = mx + b$ *such that* $f(1) = 3$ *and* $f(2) = -4$.

Exercise 408
The following shows a situation of how a linear function can be found from a verbal description.
The relationship between Celsius (C) *and Fahrenheit* (F) *degrees for measuring temperature is linear. Find an equation relating* C *and* F *if* $0°C$ *corresponds to* $32°F$ *and* $100°C$ *corresponds to* $212°F$.

Exercise 409
Each Sunday a newspaper agency sells x *copies of a newspaper for* \$1.00. *The cost to the agency of each newspaper is* \$0.50. *The agency pays a fixed cost for storage, delivery, and so on, of* \$100 *per Sunday. Find an equation that relates the profit* P, *in dollars, to the number* x *of copies sold.*

Exercise 410
In 1997, Florida Power and Light Company supplied electricity to residential customers for a monthly customer charge of \$5.65 *plus* 6.543 *cents per kilowatt-hour supplied in the month for the first 750 kilowatt-hour used. Write an equation that relates the monthly charge* C, *in dollars, to the number* x *of kilowatt-hours used in the month.*

Exercise 411
A new Toyota RAV4 costs \$21,000. *The car's value depreciates linearly to* \$10,500 *in three years time. Write a formula which expresses its value,V, in terms of its age,* t, *in years.*

Exercise 412
If a table of data represents a linear function then one can use two points of the data to find the slope m *and one point to find the y-intercept* b.
A grapefruit is thrown into the air. Its velocity, v, *is proportional to ,* t, *the time since it was thrown. A positive velocity indicates that the grapefruit is rising and a negative velocity indicates it is falling. Check that the data in the following table corresponds to a linear function. Find a formula for* v *in terms of* t.

t, time (sec)	*1*	*2*	*3*	*4*
v, velocity (ft/sec)	*48*	*16*	*-16*	*-48*

4.2 Systems of Linear Equations

To find the coordinates of the point of intersection of two lines requires solving a system of two linear equations in two unknowns. This can be done in several ways: by graphing, by substitution, or by elimination.

Exercise 413
Because the graph of a linear equation is a straight line, points that satisfy both equations lie on both lines. For some systems these points can be found by graphing. Solve the following system by graphing:

$$\begin{cases} x - y = -2 \\ x + y = 4 \end{cases}$$

Exercise 414
Solve the system by graphing:

$$\begin{cases} 2x - 3y = 6 \\ -2x + 3y = 3 \end{cases}$$

Exercise 415
Solve the system by graphing:

$$\begin{cases} x - 2y = 4 \\ 2(y + 2) = x \end{cases}$$

Exercise 416
A different way for solving a linear system is by substitution. In this method, we replace a variable in one equation with an equivalent expression obtained from the other equation. This way one eliminates a variable and get an equation involving only one variable. Solve the system by substitution

$$\begin{cases} 4x + y = 5 \\ x + 2y = -4 \end{cases}$$

Exercise 417
Solve the system by substitution:

$$\begin{cases} x - 2y = 3 \\ 2x - 4y = 7 \end{cases}$$

Exercise 418
Solve the system by substitution:

$$\begin{cases} 2x + 3y = 5 + x + 4y \\ x - y = 5 \end{cases}$$

Exercise 419
A famous national appliance store sells both Sony and Sanyo stereos. Sony sells for \$280*, and Sanyo sells for* \$315*. During a one day sale, a total of 85 Sonys and Sanyos were sold for a total of* \$23,975*. How many of each brand were sold during this one-day sale?*

Exercise 420
A third way for solving linear systems is by the method of elimination. In this method we eliminate a variable by adding equations. Solve the system by elimination:

$$\left\{\begin{array}{rcrcr} 2x & - & 3y & = & -7 \\ 3x & + & y & = & -5 \end{array}\right.$$

Exercise 421
Solve the system by elimination:

$$\left\{\begin{array}{rcrcr} x & - & 2y & = & 3 \\ -2x & + & 4y & = & 1 \end{array}\right.$$

Exercise 422
Solve the system

$$\left\{\begin{array}{rcrcr} 2x & - & y & = & 1 \\ 4x & - & 2y & = & 2 \end{array}\right.$$

Exercise 423
Find k such that the following system is inconsistent, i.e. has no solutions.

$$\left\{\begin{array}{rcrcr} 4x & - & 8y & = & -3 \\ 2x & + & ky & = & 16 \end{array}\right.$$

Exercise 424
Tickets to the Fisher Theater cost $20 *for the floor and* $16 *for the balcony. If the receipts from the sale of 1420 tickets were* $26,060*, how many tickets were sold at each price?*

Exercise 425
Solve the system

$$\left\{\begin{array}{rcrcr} \frac{1}{x} & + & \frac{1}{y} & = & 8 \\ \frac{3}{x} & - & \frac{5}{y} & = & 0 \end{array}\right.$$

Exercise 426
Find A and B such that: $\frac{6}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$.

4.3 Quadratic Functions

Exercise 427
A quadratic function is any function of the form $f(x) = ax^2 + bx + c$ with $a \neq 0$. Its graph is a curve known as **parabola.** *By using the method of completing the square we can rewrite $f(x)$ as $f(x) = a(x-h)^2 + k$. The point (h, k) is called the* **vertex** *of the parabola.*
Find the vertex of $f(x) = -3x^2 + 6x + 1$.

Exercise 428
By setting $a(x-h)^2+k=0$ *and solving for* x *we find the* $x-$*intercepts of the graph, that is the points where the graph crosses the x-axis.*
Find the x-intercepts of the function $f(x)=x^2-8x+16$.

Exercise 429
If $a>0$ *then the parabola opens up and so the vertex is the minimum point on the curve. If* $a<0$ *then the parabola opens down and the vertex is the minimum point.*
Determine the extreme point of $f(x)=-2x^2+8x+3$ *and state whether it is a minimum or a maximum point.*

Exercise 430
To sketch a parabola we look for the vertex, the x-intercepts (if any) and the y-intercept. Sketch the graph of $f(x)=2x^2+3x-2$.

Exercise 431
Sketch the graph of $f(x)=-3x^2+6x+1$.

Exercise 432
Sketch the graph of $f(x)=x^2-6x+9$.

Exercise 433
Sketch the graph of $f(x)=2x^2+x+1$.

Exercise 434
If the graph of $f(x)=ax^2+2x+3$ *contains the point* $(1,-2)$, *what is* a?

Exercise 435
If the graph of $f(x)=x^2+bx+1$ *has an x-intercept at* $x=-2$, *what is* b?

Exercise 436
Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p *dollars, the revenue* R *is*

$$R(p)=-4p^2+4,000p.$$

What unit price should be established for the dryer to maximize revenue?

Exercise 437
What is the largest rectangular area that can be enclosed with 400 ft of fencing?

Exercise 438
A farmer with 400 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?

Exercise 439
A projectile is fired from a cliff 200 ft above the water at an inclination of $45°$ *to the horizontal, with an initial velocity of* 50 *ft per second. The height* h *of the projectile above the water is given by*

$$h(x) = -\frac{32}{2500}x^2 + x + 200$$

where x *is the horizontal distance of the projectile from the base of the cliff. How far from the base of the cliff is the height of the projectile a maximum?*

Exercise 440
Find two numbers whose sum is 7 *and whose product is maximum.*

Exercise 441
An object is thrown upward from the top of a 64-foot tall building with an initial velocity of 48 feet per second. What is its maximum altitude?

4.4 Composition of functions

Exercise 442
If $f(x)$ *and* $g(x)$ *are two functions such that the range of* $g(x)$ *is contained in the domain of* $f(x)$ *then we can define a new function, denoted by* $f(g(x))$, *with domain the domain of* $g(x)$. *This function, is obtained by replacing the letter* x *in the formula of* $f(x)$ *by the expression* $g(x)$.
Let $f(x) = \frac{1}{x-1}$ *and* $g(x) = \frac{2}{x}$. *Find* $f(g(x))$.

Exercise 443
Let $f(x) = x^2$ *and* $g(x) = -|x|$. *Calculate* $g(f(-3))$.

Exercise 444
Sometimes, one needs to recognize how a function can be expressed as the composition of two functions. In this way, we are "decomposing" the function. Find functions g *and* h *such that* $h(g(x)) = \sqrt{x} - 1$.

Exercise 445
Let $f(x) = x^2 - 1$ *and* $g(x) = \sqrt{3-x}$. *Find the domain of* $f(g(x))$

Exercise 446
Let $f(x) = \frac{1}{x}, g(x) = x^2 + 1$, *and* $h(x) = x$. *Find* $f(g(h(x)))$.

Exercise 447
Find three functions f, g, *and* h *such that* $f(g(h(x))) = \frac{1}{|x|+3}$.

Exercise 448
Let $f(x) = (2x-3)^2$. *Find* g *and* h *such that* $g(h(x)) = f(x)$.

Exercise 449
Let $f(x) = \frac{1}{x-1}$ *and* $g(x) = \frac{4}{x}$. *Find the domain of* $f(g(x))$.

Exercise 450
Let $\lfloor x \rfloor$ *denote the largest integer less than or equal to* x. *For example,* $\lfloor -3 \rfloor = -3, \lfloor -3.2 \rfloor = -4, \lfloor 2.5 \rfloor = 2$. *Calculate* $\lfloor \pi \rfloor$.

Exercise 451
Graph the function $f(x) = \lfloor x \rfloor$.

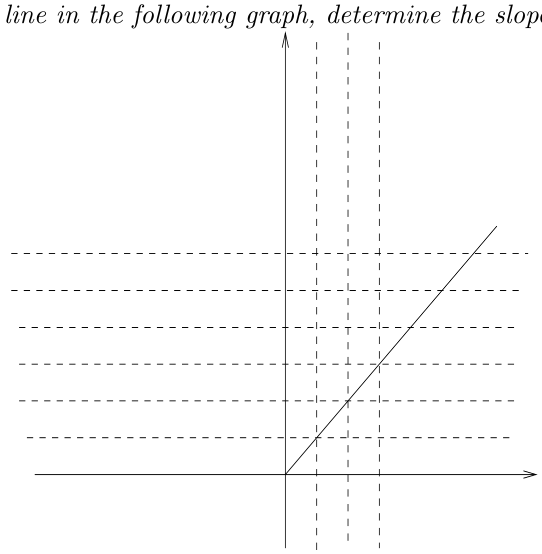
Exercise 452
Let $f(x) = \lfloor x \rfloor$ *and* $g(x) = \pi$. *Find* $f(g(x))$.

Exercise 453
What is the range of $\lfloor x \rfloor$ *?*

Exercise 454
Let $g(x) = \frac{1+x^2}{1+x^4}$ *and* $F(x) = \frac{1+x^4}{1+x^2}$. *Find* f *such that* $f(g(x)) = F(x)$.

4.5 Chapter Test

Exercise 455
For the straight line in the following graph, determine the slope.



Exercise 456
Determine the slope of the line that has the following equation:

$$2y = 2x + 2y - 6$$

Exercise 457
A line passes through the points $(-1, 5)$ *and* $(0, 4)$. *Determine the slope of the line.*

Exercise 458
Determine the slope m and the y-intercept b of the line $5x + 3y - 12 = 0$.

Exercise 459
Consider the line: $y = -2x + 9$. Suppose that x is decreased by 100. What is the change in the value of y?

Exercise 460
Write the function $y = 2x^2 + 6x - 10$ in the form $y = a(x - h)^2 + k$.

Exercise 461
Determine the equation of a quadratic function whose graph passes through the point $(1, -2)$ and has vertex $(4, 5)$

Exercise 462
A company manufactures upholstered chairs. If it manufactures x chairs, then the revenue $R(x)$ per chair is given by the function:

$$R(x) = 200 - 0.05x$$

How many chairs should the company manufacture to maximize total revenue?

Exercise 463
Let $f(x)$ be the function whose graph is obtained by translating the graph of $g(x) = 2x^2 - 3x + 1$ two units to the left and 5 units down. Find the expression of $f(x)$.

Exercise 464
Suppose that an architect wishes to design a house with a fenced backyard. To save fencing cost, he wishes to use one side of the house to border the yard and use cyclone fence for the other three sides. The specifications call for using 100 ft of fence. What is the largest area that the yard can contain?

Exercise 465
Find the maximum value: $f(x) = -5(x - 1)^2 + 3$

Exercise 466
Find the minimum value: $f(x) = 12(x - 5)^2 + 2$

Exercise 467
Let x be the amount (in hundreds of dollars) a company spends on advertising, and let P be the profit, where $P = 230 + 20x - 0.5x^2$. What expenditure for advertising results in the maximum profit?

Exercise 468
Let $f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$. Find the domain of the composition function $f(g(x))$.

Exercise 469
Which of the following pair of functions yield $f(g(x)) = g(f(x))$?

(A) $f(x) = x + 2, g(x) = 4 - x^2$ (B) $f(x) = 2x + 3, g(x) = \frac{1}{2}(x - 3)$ (C) $f(x) = 2x - 5, g(x) = 5$ (D) $f(x) = x^2 + 5, g(x) = \sqrt{1 - x}$

Exercise 470
Let $f(x) = \sqrt{x}$ *and* $g(x) = \frac{x+2}{x-2}$. *Find the domain of* $f(g(x))$.

Exercise 471
Let $f(x) = \frac{1}{x^2-1}$ *and* $g(x) = x + 1$. *Determine the domain of* $f(g(x))$.

Exercise 472
Let $f(x) = |x|$. *Find* $f(f(x))$.

Exercise 473
Express the function $f(x) = \sqrt[3]{x^2 + 1}$ *as a composition of two functions* $g(x)$ *and* $h(x)$.

Exercise 474
The number of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 55, \quad 2 \leq T \leq 14$$

where T *is the Celsius temperature of the food. When the food is removed from refrigeration, the temperature is given by*

$$T(t) = 4t + 2, \quad 0 \leq t \leq 3$$

where t *is the time in hours. The composition function* $N(T(t))$ *represents the number of bacteria as a function of the amount of time the food has been out of refrigeration. Find a formula for* $N(T(t))$

Exercise 475
Let $f(x) = 0.003 - (1.246x + 0.37)$.

(a) Calculate the average rates of change:
(i) $\frac{f(2)-f(1)}{2-1}$ *(ii)* $\frac{f(1)-f(2)}{1-2}$ *(iii)* $\frac{f(3)-f(4)}{3-4}$.
(b) Rewrite $f(x)$ *in the form* $f(x) = b + mx$.

Exercise 476
Given data from a linear function. Find a formula for the function.

t	1.2	1.3	1.4	1.5
f(t)	0.736	0.614	0.492	0.37

4.6 Cumulative Test

Exercise 477
Perform the indicated operations and simplify the result:

$$\frac{\frac{6}{a^2+3a-10}-\frac{1}{a-2}}{\frac{1}{a-2}+1}$$

Exercise 478
Reduce the following fraction to lowest terms:

$$\frac{x(x-2)-3}{(x-2)(x+1)}$$

Exercise 479
Factor completely: $8x^3+27y^3$.

Exercise 480
Factor and simplify: $(1-3x)(3x+4)^{-\frac{1}{2}}+(3x+4)^{\frac{1}{2}}$

Exercise 481
Rewrite so that all exponents are positive:

$$\frac{x^{-1}-y^{-1}}{x-y}$$

Exercise 482
Rationalize the numerator:

$$\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}$$

Exercise 483
Simplify: $\sqrt{(x-1)^4}$

Exercise 484
Expand and simplify: $(\sqrt[3]{x}-\sqrt[3]{2})(\sqrt[3]{x^2}+\sqrt[3]{2x}+\sqrt[3]{4})$

Exercise 485
Perform the indicated operation and simplify the result:

$$\frac{2}{x-2}+\frac{3}{x+1}-\frac{x-8}{x^2-x-2}$$

Exercise 486
Factor: $t(a-b)-r(b-a)$

Exercise 487
The cost of tuition at a certain college is currently \$11,000 *a year. Assume that the cost increases 8 percent each year. What will be the cost of a five-year stay at the college?*

Exercise 488
Suppose that a savings account earns interest at a 4 percent annual rate and that the interest is compounded monthly. The initial balance is $\$5,000$. *What is the balance in the account after 2 years?*

Exercise 489
Determine the following product: $(\sqrt{x-1}-2)(\sqrt{x-1}+2)$

Exercise 490
A company produces a certain type of shirts. If the price x *dollars for each shirt, then the number actually sold (in thousands) is given by the expression:*

$$N = 57 - x$$

How much revenue does the company get by selling shirts at x *dollars?*

Exercise 491
Solve the inequality: $2x^3 + x^2 - x < 0$.

Exercise 492
Solve the inequality: $-6 < \frac{2-3x}{2} \leq 13$.

Exercise 493
The formula $h(t) = -16t^2 + 64t + 960$ *gives the height* h *of an object thrown upward from the roof of a* 960 *ft building at an initial velocity of* $64ft/sec$. *For what times* t *will the height be greater than* $992ft$?

Exercise 494
Starting salaries of programmers were $\$25,000$ *in 1985 and* $\$35,000$ *in 1995. Determine a mathematical model that describes the growth of starting salaries for programmers.*

Exercise 495
Determine the equation of a line passing through the points $(5,3)$ *and* $(2,-2)$.

Exercise 496
Determine the equation of a line passing through the point $(3,-2)$ *and having slope* 4

Exercise 497
In 1999, the population of a town was $18,310$ *and growing by 58 people per year. Find a formula for,*P*, the town's population, in terms of ,*t*, the number of years since 1999.*

Exercise 498
Match the following functions to the lines.

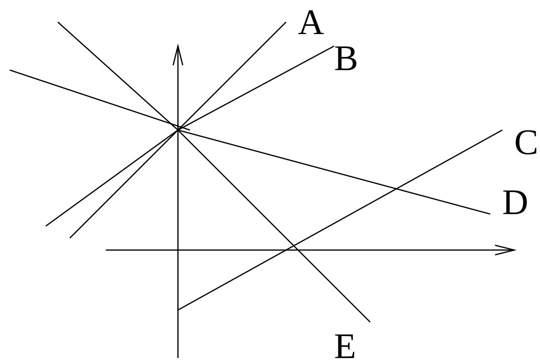
f(x) = 5+2x

g(x) = -5+2x

h(x) = 5+3x

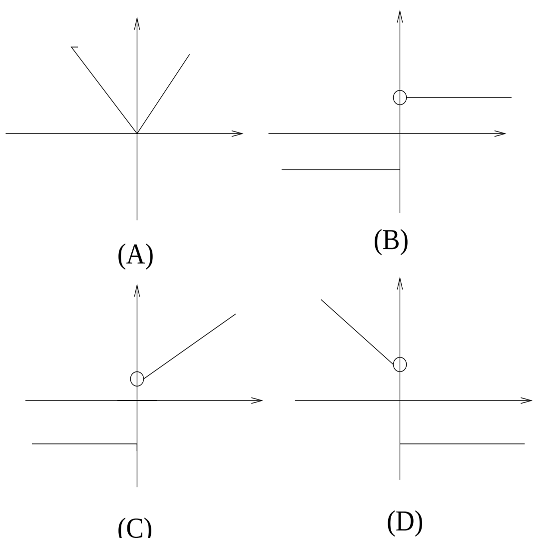
u(x) = 5-2x

v(x) = 5-3x



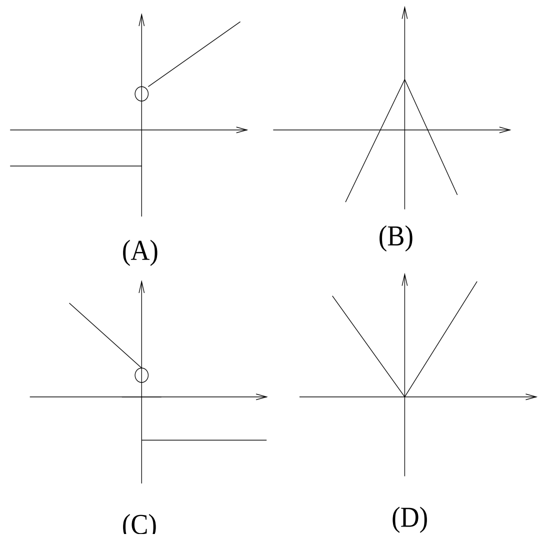
Exercise 499
Which graph corresponds to the piecewise defined function:

$$f(x) = \begin{cases} -1 & x \leq 0 \\ x+1 & x > 0 \end{cases}$$



Exercise 500
Which graph corresponds to the piecewise defined function:

$$f(x) = \begin{cases} x+1 & x \leq 0 \\ -x+1 & x > 0 \end{cases}$$


(A)
(B)
(C)
(D)

Chapter 5

Polynomial and Rational Functions

A **polynomial** function is a function that can be defined using a polynomial expression. A **rational** function is defined to be a quotient of two polynomial functions. We will study both kinds of functions and examine the zeros of these functions. We will also study the graphs of these functions.

5.1 Division Algorithms and the Remainder Theorem

An important tool for finding the zeros of polynomials is the use of division algorithm:

$$f(x) = q(x)g(x) + r(x)$$

where $q(x)$ is the quotient and $r(x)$ is the remainder. Note that $deg(r(x)) < deg(g(x))$. If $r(x) = 0$ we say that $g(x)$ is a factor of $f(x)$.

Exercise 501
Find the quotient and the remainder of the division of $f(x) = 3x^3 - 2x + 4$ *by* $g(x) = x^2 + x + 1$.

Exercise 502
Find the quotient and the remainder of the division of $f(x) = 2x^5 - x^4 + 2x^2 - 1$ *by* $g(x) = x^3 - x^2 + 1$.

Exercise 503
The Remainder Theorem states that the remainder of the division of a polynomial $f(x)$ *is* $f(c)$.
Use the Remainder Theorem to find the remainder of the division of $f(x) = x^4 + x^3 - 7x - 10$ *by* $x - 2$.

Exercise 504
Let $f(x) = x^5 + 2x^3 - x + 1$*. Use the Remainder Theorem and synthetic division to compute* $f(-2)$*.*

Exercise 505
Use the Remainder Theorem and synthetic division to determine whether the given number is a solution to the equation: $x^3 - 3x^2 + 3x - 2 = 0$*.*

Exercise 506
Without synthetic division, find the quotient and the remainder when $x^3 - a^3$ *is divided by* $x - a$*.*

Exercise 507
Find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $6x^4 + 5x^3 - 3x^2 + x + 3$ *is divided by* $2x^2 - x + 1$*.*

Exercise 508
Let $f(x) = 5x^4 - 2x^2 - 1$*. Use the Remainder Theorem and synthetic division to compute* $f(\frac{1}{3})$*.*

Exercise 509
Use the Remainder Theorem and synthetic division to determine whether the given number is a solution to the equation: $2x^3 - x^2 + 1 = 0$*.*

Exercise 510
Without synthetic division, find the quotient and the remainder when $x^3 + a^3$ *is divided by* $x + a$*.*

Exercise 511
Find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $3x^3 - 8x - 4$ *is divided by* $x^2 + 2x + 1$*.*

Exercise 512
Find k *such that when* $2x^3 + x^2 - 5x + 2k$ *is divided by* $x + 1$ *the remainder is 6.*

Exercise 513
Find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $2x^5 - 3x^3 + 2x^2 - x + 3$ *is divided by* $x^3 + 1$*.*

Exercise 514
Use synthetic division to determine $f(-3)$ *where* $f(x) = -x^4 - 5x^3 + 4x^2 - 9x + 10$*.*

Exercise 515
A ball is thrown upward from the top of a 300 feet tall building with an initial velocity of 30 feet per second. Use the Remainder Theorem and synthetic division to compute its height above the ground after 2 seconds.

5.2 The Factor Theorem and the Real Zeros of Polynomials

Exercise 516
The Factor Theorem states that if $f(c) = 0$ *then we say that* c *is a zero of the function* $f(x) = 0$ *and* $(x-c)$ *is a factor of* $f(x)$. *Candidates for zeros are ratios of the form* $\pm\frac{a}{b}$ *where* a *is a divisor of the constant term of the polynomial and* b *is a divisor of the leading coefficients. We refer to this as the Rational Zero Test.*
Find the real zeros of $f(x) = 6x^3 - 4x^2 + 3x - 2$.

Exercise 517
If a zero of a ploynomial occurs n *times then we call* n *the* **multiplicity**. *Let* $f(x) = (x-1)^3(x+5)^5$. *Find the multiplicity of the solution* -5.

Exercise 518
Determine the number a *so that* $x - 2$ *is a factor of* $x^4 + x^3 - 3x^2 + ax + a$.

Exercise 519
For which positive integers n *is* $x + a$ *a factor of* $x^n + a^n$, *where* $a \neq 0$.

Exercise 520
Use synthetic division to find $q(x)$ *such that* $x^4 - 64 = (x-2)q(x)$.

Exercise 521
Given that $f(1) = 0$ *where* $f(x) = x^3 - x^2 - 4x + 4$. *Factor* $f(x)$ *completely.*

Exercise 522
Given that $f(5) = 0$ *where* $f(x) = x^3 - 5x^2 - 5x + 25$. *Factor* $f(x)$ *completely.*

Exercise 523
Form a polynomial having the specified zeros with the specified multiplicities: 0, 1, and -3 *of multiplicities 2, 2, and 1 respectively.*

Exercise 524
Given that 3 is a zero of $f(x) = x^3 - 5x^2 + 8x - 6$. *Factor* $f(x)$ *completely.*

Exercise 525
Given that -1 *is a zero of* $f(x) = x^3 + 4x^2 - 7x - 10$. *Factor* $f(x)$ *completely.*

Exercise 526
Find the three zeros of $f(x) = x^3 - x^2 - 2x$ *by factoring.*

Exercise 527
Find a polynomial having the zeros $-3, 2,$ *and 5.*

Exercise 528
Find $q(x)$ *such that* $6x^3 - 23x^2 - 6x + 8 = (x-4)q(x)$.

Exercise 529
Given that 2 is a zero to the equation: $x^3 - 7x + 6 = 0$. *Find the remaining zeros.*

Exercise 530
Use the Rational Zero Test to list all possible rational zeros of

$$f(x) = x^3 + 3x^2 - x - 3$$

5.3 Graphs of Polynomial Functions

Exercise 531
A function is said to be **even** *if its graph is symmetric about the y-axis. Which of the following properties implies that* $f(x)$ *is even.*
(A) $f(-x) = f(x)$ *(B)* $f(-x) = -f(x)$ *(C)* $f(-x) = -f(-x)$ *(D)* $f(-x) = 2$.

Exercise 532
A function is said to be **odd** *if its graph is symmetric about the origin. Which of the following properties implies that* $f(x)$ *is odd.*

(A) $f(-x) = f(x)$ *(B)* $f(-x) = -f(x)$ *(C)* $f(-x) = -f(-x)$ *(D)* $f(-x) = 2$

Exercise 533
Which of the following functions is even.

(A) $f(x) = x^4 - 6x$ *(B)* $f(x) = x^4 + 6x$ *(C)* $f(x) = x^4 - 6x^2 + 3$ *(D)* $f(x) = x^3 + 3x$

Exercise 534
The graph of $f(x) + c, c > 0$ *is obtained from the graph of* $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward (C) by a horizontal shift, c units to the right (D) by a horizontal shift, c units to the left.

Exercise 535
The graph of $f(x) - c, c > 0$ *is obtained from the graph of* $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward (C) by a horizontal shift, c units to the right (D) by a horizontal shift, c units to the left.

Exercise 536
The graph of $f(x + c), c > 0$ *is obtained from the graph of* $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward (C) by a horizontal shift, c units to the right (D) by a horizontal shift, c units to the left.

Exercise 537
The graph of $f(x-c), c>0$ is obtained from the graph of $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward (C) by a horizontal shift, c units to the right (D) by a horizontal shift, c units to the left.

Exercise 538
The graph of $cf(x), c>1$ is

(A) is horizontal compression of the graph of $f(x)$.
(B) is a horizontal stretch of the graph of $f(x)$.
(C) is a vertical compression of the graph of $f(x)$.
(D) is a vertical stretch of the graph of $f(x)$.

Exercise 539
The graph of $cf(x), 0<c<1$ is

(A) is horizontal compression of the graph of $f(x)$.
(B) is a horizontal stretch of the graph of $f(x)$.
(C) is a vertical compression of the graph of $f(x)$.
(D) is a vertical stretch of the graph of $f(x)$.

Exercise 540
The graph of $f(cx), c>1$ is

(A) is horizontal compression of the graph of $f(x)$.
(B) is a horizontal stretch of the graph of $f(x)$.
(C) is a vertical compression of the graph of $f(x)$.
(D) is a vertical stretch of the graph of $f(x)$.

Exercise 541
The graph of $f(cx), 0<c<1$ is

(A) is horizontal compression of the graph of $f(x)$.
(B) is a horizontal stretch of the graph of $f(x)$.
(C) is a vertical compression of the graph of $f(x)$.
(D) is a vertical stretch of the graph of $f(x)$.

Exercise 542
The graph of $-f(x)$ is obtained from the graph of $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward
(C) by a reflection about the x-axis (D) by a reflection about the y-axis.

Exercise 543
The graph of $f(-x)$ is obtained from the graph of $f(x)$

(A) by a vertical shift, c units upward (B) by a vertical shift, c units downward (C) by a reflection about the x-axis (D) by a reflection about the y-axis.

Exercise 544
Sketch the graph of the function $f(x) = 3(x+2)(x-1)(x-4)$

Exercise 545
Sketch the graph of the function $f(x) = (x-2)^2(x+3)$

Exercise 546
Sketch the graph of the function $f(x) = x^4 - 4x^2$

Exercise 547
Sketch the graph of the function $f(x) = (x-2)^3 + 1$

Exercise 548
Sketch the graph of the function $f(x) = -x^4$

Exercise 549
Sketch the graph of the function $f(x) = x^4 - 4x^3 - 4x^2 + 16x$

Exercise 550
Let $f(x) = x^2, g(x) = x^2-2$, *and* $h(x) = x^2+3$. *What is the relationship between the graph of* $f(x)$ *and the graphs of* $g(x)$ *and* $h(x)$?

Exercise 551
At a jazz club, the cost of an evening is based on a cover charge of \$5 plus a beverage charge of \$3 per drink.

(a) Find a formula for $t(x)$, *the total cost for an evening in which* x *drinks are consumed.*
(b) If the price of the cover charge is raised by \$1, express the new total cost function, $n(x)$, *as a transformation of* $t(x)$.
(c) The management decides to increase the cover charge to \$10, leave the price of a drink at \$3, but include the first two drinks for free. For $x \geq 2$, *express* $p(x)$, *the new total cost, as a transformation of* $t(x)$.

Exercise 552
Sketch the graphs of $f(x) = -x^2$ *and* $y = f(-x)$ *on the same set of axes. How are these graphs related? Give an explicit formula for* $y = f(-x)$.

Exercise 553
Sketch the graphs of $y = g(x) = -x^2$ *and* $y = -g(x)$ *on the same set of axes. How are these graphs related? Give an explicit formula for* $y = -g(x)$.

Exercise 554
Are the following functions even, odd, or neither?

(A) $f(x) = \frac{1}{x^2}$ *(B)* $g(x) = x^3 + x$ *(C)* $h(x) = x^2 + 2x$ *(D)* $j(x) = 2^{x+1}$

Exercise 555
Let $f(x) = x^3$.

(a) Sketch the graph of the function obtained from f *by first reflecting about the x-axis, then translating up two units. Write a formula for the resulting function.*
(b) Sketch the graph of the function obtained from f *by first translating up two units, then reflecting about x-axis. Write a formula for the resulting function.*
(c) Are the functions you found in parts (a) and (b) the same?

Exercise 556
A company projects a total profit, $P(t)$ *dollars, in year* t*. Explain the economic meaning of* $r(t) = 0.5P(t)$ *and* $s(t) = P(0.5t)$.

Exercise 557
You are a banker with a table showing year-end values of \$1 *invested at an interest rate of* 1% *per year, compounded annually, for a period of 50 years.*

(a) Can this table be used to show 1% *monthly interest charges on a credit card? Explain.*
(b) Can this table be used to show values of an annual interest rate of 5%*? Explain.*

5.4 Graphs of Rational Functions

Exercise 558
A **rational function** *is a function that is the quotient of two polynomials* $\frac{f(x)}{g(x)}$.
The domain consists of all numbers such that $g(x) \neq 0$.
Find the domain of the function $f(x) = \frac{x-2}{x^2-x-6}$.

Exercise 559
Geometrically, the values of x *for which the denominator of a rational function is zero are called* **vertical asymptotes**. *Thus, if* $x = a$ *is a vertical asymptote then as* x *approaches* a *from either sides the function becomes either positively large or negatively large. The graph of a function never crosses its vertical asymptotes. Find the vertical asymptote of the function* $f(x) = \frac{2x-11}{x^2+2x-8}$

Exercise 560
If $f(x)$ *approaches a value* b *as* $x \to \infty$ *or* $x \to -\infty$ *then we call* $y = b$ *a* **horizontal asymptote**. *The graph of a rational function may cross its horizontal asymptote. Find the horizontal asymptote, if it exists, for each of the following functions:*

(a) $f(x) = \frac{3x^2+2x-4}{2x^2-x+1}$.
(b) $f(x) = \frac{2x+3}{x^3-2x^2+4}$.
(c) $f(x) = \frac{2x^2-3x-1}{x-2}$.

Exercise 561
If $\lim_{x\to\pm\infty}((mx+b)-f(x))=0$ *then we call the line* $y=mx+b$ *an* **oblique asymptote.** *Find the oblique asymptote of the function* $f(x)=\frac{2x^2-3x-1}{x-2}$.

To graph a rational function $h(x)=\frac{f(x)}{g(x)}$:

1. Find the domain of $h(x)$ and therefore sketch the vertical asymptotes of $h(x)$.
2. Sketch the horizontal or the oblique asymptote if they exist.
3. Find the $x-intercepts$ of $h(x)$ by solving the equation $f(x)=0$.
4. Find the y-intercept: $h(0)$
5. Draw the graph

Exercise 562
Sketch the graph of the function $f(x)=\frac{1}{x^2}$

Exercise 563
Sketch the graph of the function $f(x)=\frac{2}{x+3}$

Exercise 564
Sketch the graph of the function $f(x)=\frac{-3}{(x-1)^2}$

Exercise 565
Sketch the graph of the function $f(x)=\frac{3x}{x+1}$

Exercise 566
Sketch the graph of the function $f(x)=\frac{x}{x^2-1}$

Exercise 567
Find the vertical asymptotes of $f(x)=\frac{x}{x^2+x-2}$

Exercise 568
Find the horizontal asymptote of $f(x)=\frac{x^2}{x^2+x-2}$

Exercise 569
Find the oblique asymptote of $f(x)=\frac{x^2-1}{2x}$

Exercise 570
Sketch the graph of the function $f(x)=\frac{4}{x^2+1}$

Exercise 571
Sketch the graph of the function $f(x)=\frac{2x+1}{x+1}$

Exercise 572
Sketch the graph of the function $f(x)=\frac{2x^2}{3x^2+1}$

Exercise 573
Write a rational function with vertical asymptotes $x=-2$ *and* $x=1$.

Exercise 574
Find the horizontal asymptote of $f(x) = \frac{2x-1}{x^2+1}$

Exercise 575
Find the zeros of the rational function $f(x) = \frac{x^2+x-2}{x+1}$.

Exercise 576
Find the y-intercept of the function $f(x) = \frac{3}{x-2}$.

5.5 Chapter Test

Exercise 577
Find the oblique asymptote of $f(x) = \frac{2x^3-1}{x^2-1}$

Exercise 578
Write a rational function satisfying the following criteria:
Vertical asymptote: $x = -1$.
Horizontal asymptote: $y = 2$.
x-intercept: $x = 3$.

Exercise 579
Sketch the graph of the function $f(x) = \frac{x^2-x-2}{x-1}$.

Exercise 580
Find the domain of the function $f(x) = \frac{x+4}{x^2+x-6}$

Exercise 581
Find the horizontal asymptote of $f(x) = \frac{x^2}{3x^2-4x-1}$

Exercise 582
Sketch the graph of the function $f(x) = x^3 - 2x^2 + x - 1$.

Exercise 583
Sketch the graph of the function $f(x) = x^5 - x$.

Exercise 584
Sketch the graph of the function $f(x) = -2x^4 + 2x^2$.

Exercise 585
For the polynomial

$$f(x) = 5(x-2)(x+3)^2(x-\frac{1}{2})^3(x+\frac{1}{2})^4$$

which zero is of multiplicity 3?
(A) $-\frac{1}{2}$ *(B)* -3 *(C)* 2 *(D)* $\frac{1}{2}$

Exercise 586
Determine which function is a polynomial function.

(A) $f(x) = 1 - \frac{1}{x}$ *(B)* $f(x) = \sqrt[3]{x}$ *(C)* $f(x) = x^3 - 3x^2 + 1$ *(D)* $f(x) = x^2 - \sqrt{x}$

Exercise 587
Use the Remainder Theorem and synthetic division to determine whether the given number is a zero to the equation:

$$5x^3 + 2x^2 + 5x + 2 = 0$$

(A) 0.4 *(B)* -4 *(C)* -0.4 *(D)* 4

Exercise 588
Find the quotient $q(x)$ *and the remainder* $r(x)$ *of the division of* $f(x) = 2x^3 + 3x^2 + x$ *by* $g(x) = x^3 + 1$.

Exercise 589
Find the quotient $q(x)$ *and the remainder* $r(x)$ *of the division of* $f(x) = x^4 + x^3 + x + 1$ *by* $g(x) = 5x^2 - x$.

Exercise 590
Let $f(x) = 2x^3 - x^2 + 5x + 1$. *Use the Remainder Theorem and synthetic division to compute* $f(1)$.

Exercise 591
Without synthetic division find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $x^2 - a^2$ *is divided by* $x - a$.

Exercise 592
Let $f(x) = x^3 + 2x^2 - 5x - 6$. *Use the Factor Theorem to determine whether* $x + 1$ *is a factor of* $f(x)$. *If so, what is the value of* $f(-1)$.

Exercise 593
Let $f(x)$ *be a polynomial of degree* n. *What is the maximum number of zeros of* $f(x)$?

Exercise 594
Use the Rational Zero Test to completely factor the polynomial $f(x) = 4x^4 - 4x^3 - 25x^2 + 6$.

Exercise 595
List all possible real zeros of the polynomial $f(x) = 2x^3 - 3x^2 + 2x + 2$.

Exercise 596
A plot of land has the shape of a right triangle with a hypotenuse 1 ft longer than one of the sides. Find the lengths of the sides of the plot of land if its area is 6 ft^2

5.6 Cumulative Test

Exercise 597
Determine the equation of a line passing through the point $(4, -6)$ *and parallel to the line containing the points* $(-2, 3)$ *and* $(4, 5)$.

Exercise 598
Find k *so that the line containing* $(-2, k)$ *and* $(3, 8)$ *is parallel to the line containing the points* $(5, 3)$ *and* $(1, -3)$.

Exercise 599
Sketch the graph of the parabola $f(x) = (x + 2)^2 - 3$.

Exercise 600
Write the function $f(x) = -x^2 + 6x - 8$ *in the form* $f(x) = a(x - h)^2 + k$.

Exercise 601
Find two positive real numbers whose sum is 110 *and the product is maximum.*

Exercise 602
Sketch the graph of $f(x) = \frac{x}{x^2-x-2}$.

Exercise 603
Sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$.

Exercise 604
Sketch the graph of $f(x) = \frac{x^2-x}{x+1}$.

Exercise 605
Sketch the graph of $f(x) = 1 - x^5$.

Exercise 606
Sketch the graph of $f(x) = x^3 + x^2 - 12x$.

Exercise 607
Find a cubic function f *with leading coefficient 1 and with x-intercepts* $-2, 3,$ *and* 4.

Exercise 608
Find a cubic function $f(x)$ *whose graph has x-intercepts* $-1, 0,$ *and* 2 *and also passes through* $(1, -6)$.

Exercise 609
Find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $f(x) = x^3 + x^2 - 4$ *is divided by* $g(x) = x^4 + 1$.

Exercise 610
Find the quotient $q(x)$ *and the remainder* $r(x)$ *when* $f(x) = 3x^3 - 2x^2 + 5x + 7$ *is divided by* $g(x) = 3x + 1$.

Exercise 611
Find the domain of $f(g(x))$ *if* $f(x) = \frac{1}{x+2}$ *and* $g(x) = \frac{4}{x-1}$.

Exercise 612
Solve: $|\frac{x-1}{x+1}| \leq 1$.

Exercise 613
Solve: $\frac{x+1}{1-x}$.

Exercise 614
Solve: $\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$.

Exercise 615
Solve: $|x^2 + x - 1| = 1$.

Exercise 616
Descartes Rule of sign is used to provide information about the number of real zeros of a polynomial function with real coefficients. The steps of the rule are as follows:

The number of positive real zeros of a polynomial $P(x)$ *is either:*

1. The same as the number of variations of sign in $P(x)$, *or*
2. Less than the number of variations of sign of $P(x)$ *by a positive even integer.*

The number of negative real zeros of $P(x)$ *is either:*

3. The same as the number of variations of sign in $P(-x)$, *or*
4. Less than the number of variations of sign in $P(-x)$ *by a positive even integer.*

Use Descartes' rule of signs to determine the possible number of positive zeros of the function $f(x) = 3x^6 - 4x^4 + 3x^3 + 2x^2 - x - 3$.

Exercise 617
In a college meal plan you pay a membership fee; then all your meals are at a fixed price per meal.

(a) If 30 meals cost \$152.50 *and 60 meals cost* \$250, *find the membership fee and the price per meal.*
(b) Write a formula for the cost of a meal plan, C, *in terms of the number of meals,* n.
(c) Find the cost for 50 meals.
(d) Find n *in terms of* C.
(e) Use part (d) to determine the maximum number of meals you can buy on a budget of \$300.

Exercise 618
In economics terms, the **demand** *for a product is the amount of that product*

that consumers are willing to buy at a given price. The quantity demanded of a product usually decreases if the price of that product increases. Suppose that a company believes there is a linear relationship between the demand for its product and its price. The company knows that when the price of its product was \$3 *per unit, the quantity demanded weekly was 500 units, and when the unit price was raised to* \$4*, the quantity demanded weekly dropped to 300 units. Let* D *represent the quantity demanded weekly at a unit price of* p *dollars.*

(a) Find a formula for D *in terms of* p*.*
(b) Give an economic interpretation of the slope of the function in part (a).
(c) Find D *when* $p = 0$*. Find* p *when* $D = 0$*. Give economic interpretation of both results.*

Exercise 619
In economics terms, the **supply** *for a product is the quantity of that product that suppliers are willing to provide at a given price. The quantity supplied of a product usually increases if the price of that product increases. Suppose that a company believes there is a linear relationship between the quantity supplied ,* S*, for its product and its price. The quantity supplied weekly is* \$100 *when the price is* \$2 *and the quantity supplied rises by 50 units when the price rises by* \$0.50*.*

(a) Find a formula for S *in terms of* p*.*
(b) Give an economic interpretation of the slope of the function in part (a).
(c) Is there a price below which suppliers will not provide this product?
(d) The **market clearing price** *is the price at which supply equals demand. According to theory, the free market price of a product is its market clearing price. Using the demand equation from the previous problem, find the market clearing price for this product.*

Chapter 6

Exponential and Logarithmic Functions

Recall that a function takes an input to an output. Suppose that we can reverse the process and take the output back to an input. That process produces the **inverse** of the original function. In this chapter we study the family of exponential functions and their inverses, the logarithmic functions.

6.1 One-to-one and Inverse Functions

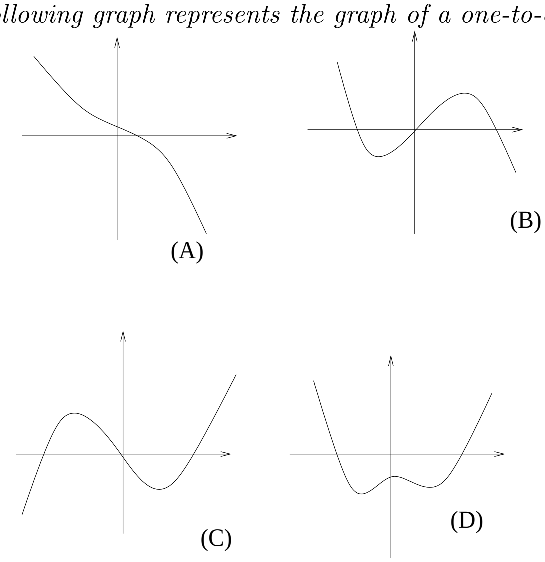
Exercise 620
If the graph of a function is such that every horizontal line intersects the graph in at most one point then we say that the function is **one-to-one**. *Show that the function $f(x) = x^3 - 5$ is one-to-one function.*

Exercise 621
If a function f is one-to-one then there exists a new function, denoted by f^{-1}, with the following propertis:

1. $y = f^{-1}(x)$ is equivalent to $f(y) = x$.
2. Dom (f^{-1}) = Range (f) and Range (f^{-1}) = Dom (f).
3. $f(f^{-1}(x)) = x$ for all x in Dom (f^{-1}) and $f^{-1}(f(x)) = x$ for all x in Dom (f).
To find a formula for f^{-1} we start by interchanging the letters x and y in $y = f(x)$ to obtain $x = f(y)$ and then we solve for y in terms of x.
Find the inverse function of $f(x) = x^3 - 5$.

Exercise 622
Geometrically, the graph of f^{-1} is the reflection of the graph of f about the line $y = x$. That is, if (x, y) is a point on the graph of f then (y, x) is a point on the graph of f^{-1}.
Find the graph of the inverse function of $f = \{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$.

Exercise 623
Which of the following graph represents the graph of a one-to-one function?



Exercise 624
Find the inverse of the function $f(x) = x^3 - 1$.

Exercise 625
Find the inverse function of $f(x) = \frac{2x+1}{x-1}$.

Exercise 626
Find the domain of $f^{-1}(x)$ *if* $f(x) = \frac{1}{x-2}$.

Exercise 627
Find the range of $f^{-1}(x)$ *if* $f(x) = \frac{4}{\sqrt{x}}$.

Exercise 628
Find the inverse function of $f(x) = 1 - 2\sqrt{x}$.

Exercise 629
Compute $f^{-1}(2)$ *if* $f(x) = \sqrt{2x}$.

Exercise 630
Find the inverse of the function $f(x) = 2x + 3$.

Exercise 631
Find the inverse of $f(x) = -|x-1|$ *for* $x \leq 1$.

Exercise 632
Find the inverse of $f(x) = \sqrt[3]{\frac{x+1}{2}}$.

Exercise 633
Which of the following is a one-to-one function?

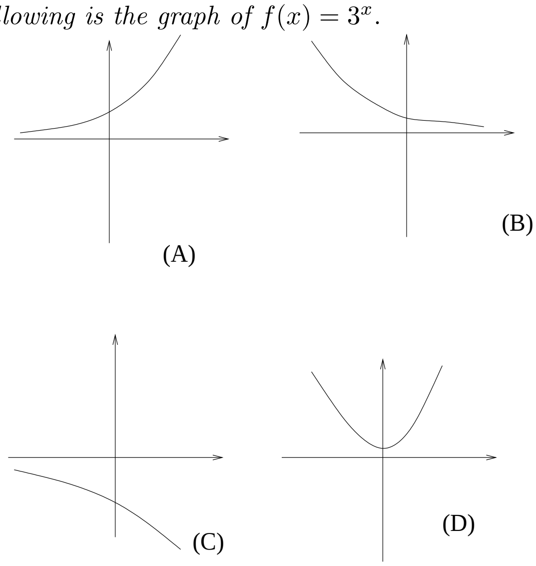
(A) $f(x) = x^3 + 1$ *(B)* $f(x) = |x|$ *(C)* $f(x) = x^2 - x$ *(D)* $f(x) = -x^2$
(E) $f(x) = -|x| + 1$

Exercise 634
Find the range of $f^{-1}(x)$ *if* $f(x) = \sqrt{2x - 3}$.

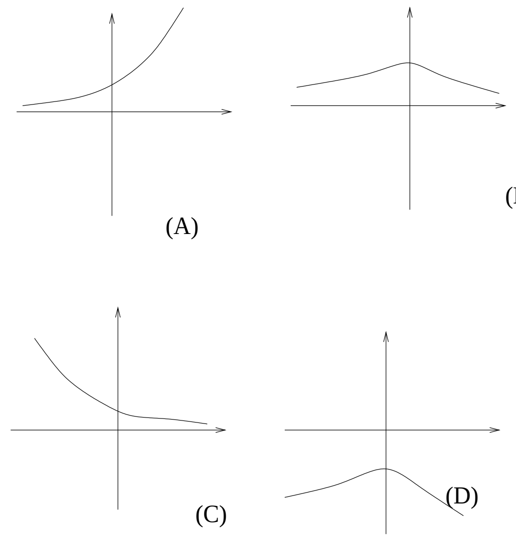
Exercise 635
Let $f(x) = \sqrt[3]{\frac{x+1}{2}}$. *Compute* $(f^{-1} \circ f^{-1})(1)$.

6.2 Exponential Functions

Exercise 636
The function $f(x) = a^x$ *where* $a > 0, a \neq 1$ *and* x *a real number is called the* **exponential function** *with* **base** a.
Which of the following is the graph of $f(x) = 3^x$.

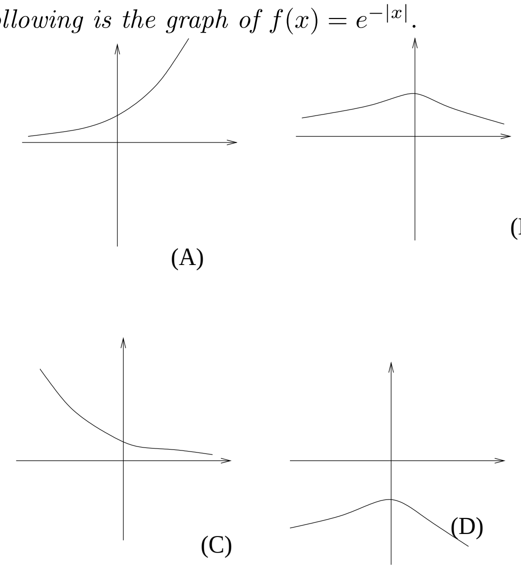


Exercise 637
Which of the following is the graph of $f(x) = 4^{-x}$.



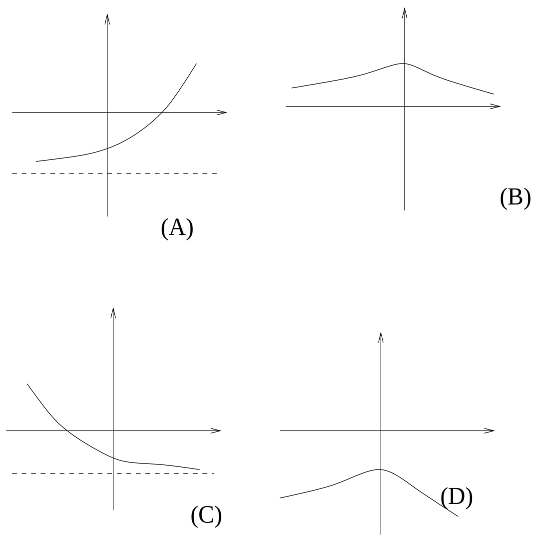
Exercise 638
For n very large the quantity $(1+\frac{1}{n})^n$ is approximated by $e \approx 2.71\cdots$. Simplify: $(e^x+e^{-x})^2-(e^x-e^{-x})^2$.

Exercise 639
Which of the following is the graph of $f(x) = e^{-|x|}$.



Exercise 640
Using a calculator evaluate $2^{1.4}$ to three decimal places.

Exercise 641
Which of the following is the graph of $f(x) = 2^{-x} - 3$.



Exercise 642
What is the horizontal asymptote of $f(x) = 2^{-x} - 3$?

Exercise 643
What is the horizontal asymptote of $f(x) = -e^{x-3}$?

Exercise 644
Sketch the graph of the piecewise defined function

$$f(x) = \begin{cases} e^{-x}, & x < 0 \\ e^{x}, & x \leq 0 \end{cases}$$

Exercise 645
If $2^x = 3$ *what does* 4^{-x} *equal?*

Exercise 646
Suppose that P *dollars are deposited in a bank account paying annual interest at a rate* r *and compounded* n *times per year. After a length of time* t, *in years, the amount* $A(t)$ *in the account is given by the formula*

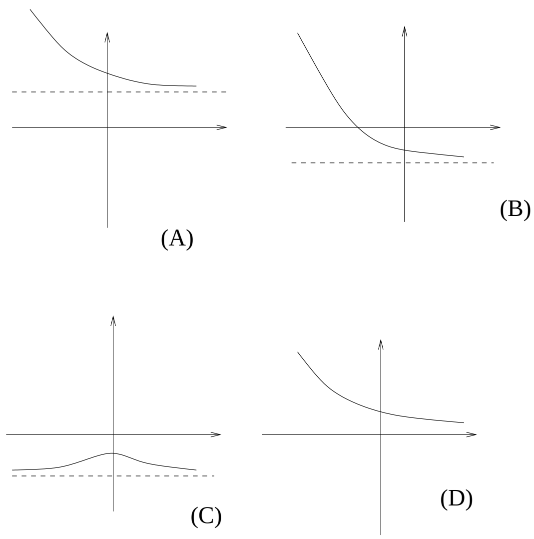
$$A(t) = P(1 + \frac{r}{n})^{nt}$$

Suppose that $n = 1$, $r = 10\%$, $P = \$10,000$ *and* $t = 10$ *years. Find* $A(10)$. *Round the answer to the nearest penny.*

Exercise 647
Simplify: $(e^x + 1)(e^x - 4)$.

Exercise 648
Suppose that at time t (in hours), the number $N(t)$ of $E-coli$ bacteria in a culture is given by the formula $N(t) = 5000e^{0.1t}$. How many bacteria are in the culture at time 5 hours? Approximate the answer to a whole integer.

Exercise 649
Sketch the graph of $f(x) = e^{-x} + 2$.



Exercise 650
Simplify: $(e^x + e^{-x})(e^x - e^{-x})$.

Exercise 651 *(Hyperbolic Sine)*
The expression $\frac{e^x - e^{-x}}{2}$ *is called the* **hyperbolic sine**. *We write* $\sinh x = \frac{e^x - e^{-x}}{2}$. *Graph* $\sinh x$.

Exercise 652 *(Hyperbolic Cosine)*
The expression $\frac{e^x + e^{-x}}{2}$ *is called the* **hyperbolic cosine**. *We write* $\cosh x = \frac{e^x + e^{-x}}{2}$. *Graph* $\cosh x$.

Exercise 653 *(Hyperbolic Tangent)*
The expression $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ *is called the* **hyperbolic tangent**. *We write* $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. *Graph* $\tanh x$.

Exercise 654
Show, algebraically, that

(a) $\cosh 0 = 1$ *and* $\sinh 0 = 0$. *(y-intercepts)*
(b) $\cosh(-x) = \cosh x$. *That is,* $\cosh x$ *is an even function.*
(c) $\sinh(-x) = -\sinh x$. *That is,* $\sinh x$ *is an odd function.*

Exercise 655
Show that

(a) $\tanh x = \frac{\sinh x}{\cosh x}$.
(b) $\cosh^2 x - \sinh^2 x = 1$.
(c) $\cosh x > \sinh x$ *for all* x.

6.3 Logarithmic Functions

Exercise 656
We define $y = \log_a x$ *as that number* y *such that* $a^y = x$ *where* $x > 0, a > 0,$ *and* $a \neq 1$. *We call* $a^y = x$ *the* **exponential form** *of* $y = \log_a x$ *and* $y = \log_a x$ *the* **logarithmic form** *of* $a^y = x$.
Write the logarithmic form of $x^\pi = e$.

Exercise 657
Write the exponential form of $\log_\pi x = \frac{1}{2}$.

Exercise 658
Find the exact value of $\log_{\sqrt{3}} 9$.

Exercise 659
Find the domain of the function $f(x) = \log_5 \frac{x+1}{x}$.

Exercise 660
Find k *such that the graph of* $\log_k x$ *contains the point* $(2, 2)$.

Exercise 661
Sketch the graph of $f(x) = \ln(4 - x)$ *where* $\ln x = \log_e x$ *is the* **natural logarithm function.**

Exercise 662
Sketch the graph of $f(x) = 2 - \ln x$.

Exercise 663
Sketch the graph of

$$f(x) = \begin{cases} -\ln x & , 0 < x < 1 \\ \ln x & , x \geq 1 \end{cases}$$

Exercise 664
Find x *such that* $2 \cdot 5^x = 4$.

Exercise 665
Find x *such that* $5 \log_2 x = 20$.

Exercise 666
Sketch the graph of $f(x) = |\log_2 x|$.

Exercise 667
Find the domain of the function $f(x) = \log_2(2 - x - x^2)$.

Exercise 668
Solve for x*:* $\log_2 x = \log_2 3$.

Exercise 669
Simplify: $a^{\log_{a^2} b}$.

Exercise 670
Simplify: $e^{\ln 2 - 3\ln 5}$.

6.4 Properties of Logarithmic Functions

Exercise 671
Complete the following:

1. $\log_a(uv) =$
2. $\log_a(\frac{u}{v}) =$
3. $\log_a u^n =$
4. $\log_a 1 =$
5. $\log_a a =$
6. $a^{\log_a u} =$

Exercise 672
Let $\ln 2 = a$ *and* $\ln 3 = b$. *Write* $\ln \sqrt[4]{48}$ *in terms of* a *and* b.

Exercise 673
Write the following expression as a sum/difference of logarithms.

$$\ln \frac{5x^2 \sqrt[3]{1-x}}{4(x+1)^2}$$

Exercise 674
Write the following expression as a single logarithm.

$$\ln(\frac{x}{x-1}) + \ln(\frac{x+1}{x}) - \ln(x^2 - 1)$$

Exercise 675
Find the exact value of : $5^{\log_5 6 + \log_5 7}$.

Exercise 676
Use the change of base formula and a calculator to evaluate $\log_{\frac{1}{2}} 15$ *to three decimal places.*

Exercise 677
Simplify: $\log_a(x + \sqrt{x^2 - 1}) + \log_a(x - \sqrt{x^2 - 1})$

Exercise 678
Given: $2\ln y = -\frac{1}{2}\ln x + \frac{1}{3}\ln(x^2+1) + \ln C$. *Express* y *in terms of* x *and* C.

Exercise 679
A reservoir has become polluted due to an industrial waste spill. The pollution has caused a buildup of algae. The number of algae $N(t)$ *present per 1000 gallons of water* t *days after the spill is given by the formula:* $N(t) = 100e^{2.1t}$.
How long will it take before the algae count reaches $20,000$ *per 1000 gallons? Write answer to three decimal places.*

Exercise 680
Let $f(x) = \ln x$. *Express the difference quotient* $\frac{f(x+h)-f(x)}{h}$ *as a single logarithm.*

Exercise 681
Express the product $(\log_x a)(\log_a b)$ *as a single logarithm.*

Exercise 682
After t *years the value of a car that originally cost* $\$14,000$ *is given by:* $V(t) = 14,000(\frac{3}{4})^t$. *Find th value of the car two years after it was purchased. Write your answer to two decimal places.*

Exercise 683
On the day a child is born, a deposit of $\$50,000$ *is made in a trust fund that pays* 8.75% *interest compounded continuously. Determine the balance in the account after 35 years. Write your answer to two decimal places. Recall that* $A(t) = Pe^{rt}$.

Exercise 684
Find a constant k *such that* $\log_2 x = k\log_8 x$.

Exercise 685
Solve for $x : \log(\log_5(\log_2 x)) = 0$

Exercise 686
Write as a single logarithm: $2\log_a x - 3\log_a y + \log_a x + y$.

6.5 Solving Exponential and Logarithmic Equations

Exercise 687
Solve: $\ln(\frac{x-1}{2x}) = 4$.

Exercise 688
Solve: $2e^{2x} - e^x - 2 = 0$.

Exercise 689
Solve: $\ln(x^2+2x) = 0$.

Exercise 690
Solve: $\log_4(x+5)+\log_4(x-5)=2.$

Exercise 691
Solve: $\log_3(2x-5)-\log_3(3x+1)=4.$

Exercise 692
Solve: $\frac{\log(x+1)}{\log x}=2.$

Exercise 693
Solve: $xa^{3\log_a x}=16.$

Exercise 694
Solve: $\frac{e^x+e^{-x}}{2}=1.$

Exercise 695
Solve: $\frac{e^x+e^{-x}}{e^x-e^{-x}}=1.$

Exercise 696
Solve: $\ln(x-2)+\ln(2x-3)=2\ln x$

Exercise 697
The demand equation for a certain product is given by

$$p=500-0.5e^{0.004x}$$

Find the demand x *for the price* $p=\$350$. *Round answer to a whole integer.*

Exercise 698
Solve: $e^{x-1}=e^x.$

Exercise 699
Solve: $\log(x-2)-\log(x+2)=\log(x-1).$

Exercise 700
Solve: $4^x-2^x-12=0.$

Exercise 701
Solve: $5^{x-2}=3^{x+2}.$

6.6 Chapter Test

Exercise 702
Find the inverse of the function $f(x)=(1-x^3)^{\frac{1}{5}}+2.$

Exercise 703
What is the relationship between $(f^{-1})^{-1}$ *and* f*?*

Exercise 704
Express $(f \circ g)^{-1}(x)$ *in terms of* $f^{-1}(x)$ *and* $g^{-1}(x)$.

Exercise 705
Sketch the graph of $f^{-1}(x)$ *if* $f(x) = \sqrt{1-x^2}, 0 \leq x \leq 1$.

Exercise 706
Sketch the region bounded by $y = e^x, y = e^{-x}$ *and the line* $y = 3$.

Exercise 707
Sketch the graph of : $f(x) = 1 - 2^{-x}$.

Exercise 708
Sketch the graph of : $f(x) = |2^x - 1|$.

Exercise 709
A radioactive decay function is given by

$$Q(t) = Q_0 2^{-\frac{t}{H}}$$

where Q_0 *denotes the amount of a radioactive susbtance at time* $t = 0$ *and* H *is the half-life. Suppose that* $Q_0 = 4$ *and* $H = 5.3$. *Find* $Q(2)$. *Round answer to three decimal places.*

Exercise 710
Simplify: $\frac{(1+e^x)(1+e^{-x})-(1+e^x)(1-e^{-x})}{(1+e^x)^2}$

Exercise 711
Find a constant k *such that* $\log_8 x = k \log_{16} x$.

Exercise 712
Solve: $\pi^{1-x} = e^x$.

Exercise 713
Solve: $\log(x^2 - x + 1) + \log(x + 1) = 1$.

Exercise 714
Solve: $2^x < 3$.

Exercise 715
Solve: $e^{8x+2} = 3^x$.

Exercise 716
Find the exponential function $y = Ae^{bx}$ *that passes through the two points* $(0, 2)$ *and* $(4, 3)$.

Exercise 717
Solve: $\log_2(x^2 + 1) - \log_4 x^2 = 1$.

Exercise 718
Solve: $(\sqrt[3]{2})^{2-x} = 2^{x^2}$.

Exercise 719
What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years? Round your answer to two decimal places.

Exercise 720
How long will it take for an investment to double in value if it earns 5% *compounded continuously? Write answer to two decimal places.*

Exercise 721
One of the problems that challenged early Greek mathematicians was whether it was possible to construct a square whose area was equal to that of a given circle.

(a) If the radius of a given circle is r, *give an expression, in terms of* r, *for the side,* s, *of the square with area equal to that of the circle.*
(b) The side, s, *of such a square is a function of the radius of the circle. What kind of function is it? How do you know?*
(c) For what values of r *is the side,* s, *equal to zero?*

6.7 Cumulative Test

Exercise 722
Find the inverse $f^{-1}(x)$ *of* $f(x) = x^2 - 4, x \geq 0$.

Exercise 723
Let $f(x) = 23,457x - 3,456$ *find* $(f \circ f^{-1})(5,00023)$.

Exercise 724
The relationship between degree Fahrenheit and degree Celsius is given by $F = \frac{9}{5}C + 32$. *Write* C *in terms of* F.

Exercise 725
Sketch the region bounded by $y = e^x, y = e$ *and* $y - axis$.

Exercise 726
Solve: $\log_2(\log_x 3) = 1$.

Exercise 727
Solve: $e^{2x} - 3e^x + 2 = 0$.

Exercise 728
Find the length of a rectangle of width 3 inches if the diagonal is 20 inches long.

Exercise 729
What number does the repeating decimal 0.123123... *equal?*

Exercise 730
A ball is thrown vertically upward from the top of a building 96 feet tall with an initial velocity of 80 *feet per second. The distance* s *(in feet) of the ball from the ground after* t *seconds is* $s = 96 + 80t - 16t^2$. *After how many seconds does the ball strike the ground?*

Exercise 731
The monthly revenue achieved by setting x *wristwatches is figured to be* $x(40 - 0.2x)$ *dollars. The wholesale cost of each watch is* \$28. *How many watches must be sold each month to achieve a profit (revenue - cost) of at least* \$100?

Exercise 732
Solve: $-2(5 - 3x) + 8 = 4 + 5x$.

Exercise 733
Solve: $|2x - 3| = 5$

Exercise 734
Solve: $\frac{(x-2)(x-2)}{x-3} > 0$.

Exercise 735
Find the equation of the line perpendicular to the line $x + y - 2 = 0$ *and passing through* $(1, -3)$.

Exercise 736
Find the radius of the circle: $x^2 + y^2 - 2x + 4y - 4 = 0$.

Exercise 737
How much water should be added to 64 ounces of a 10% *salt solution to make a* 2% *salt solution?*

Exercise 738
Given that $f(x)$ *is a linear function,* $f(4) = -5$ *and* $f(0) = 3$, *write the formula for* $f(x)$.

Exercise 739
Find the domain of the function $f(x) = \frac{x}{x^2+2x-3}$.

Exercise 740
Find the domain of $(f \circ g)(x)$ *where* $f(x) = \frac{x+1}{x-1}$ *and* $g(x) = \frac{1}{x}$.

Exercise 741
Sketch the graph of $f(x) = |x| - 4$.

Exercise 742
Line l *is given by* $y = 3 - \frac{3}{2}x$ *and point* P *has coordinates* $(6, 5)$.

(a) Find the equation of the line containing P *and parallel to* l.
(b) Find the equation of the line containing P *and is perpendicular to* l.
(c) Graph the equations in parts (a) and (b).

Exercise 743
Hooke's Law states that the force in pounds, $F(x)$, necessary to keep a spring stretched x units beyond its natural length is directly proportional to x, so $F(x) = kx$. The positive constant k is called the spring constant.

(a) Do you expect $F(x)$ to be an increasing function or a decreasing function of x? Explain.
(b) A force of 2.36 pounds is required to hold a certain spring stretched 1.9 inches beyond its natural length. Find the value of k for this spring and rewrite the formula for $F(x)$ usinf this value of k.
(c) How much force is needed to stretch this spring 3 inches beyond its natural length?