37 Basic Geometric Shapes and Figures

In this section we discuss basic geometric shapes and figures such as points, lines, line segments, planes, angles, triangles, and quadrilaterals.

The three pillars of geometry are points, lines, and planes: A **point** is an undefined term used to describe for example a location on a map. A point has no length, width, or thickness and we often use a dot to represent it. Points are usually labeled with uppercase letters.

Line is another basic term of geometry. Like a point, a line is an undefined term used to describe a tightly stretched thread or a laser beam. We can say that a line is a straight arrangement of points. A line has no thickness but its length goes on forever in two directions as shown in Figure 37.1. The arrows represent the fact that the line extends in both directions forever. A line is often named by a lowercase letter such as the line k in Figure 37.1.



Figure 37.1

The subset of the line k consisting of all points between A and B together with A and B forms a **line segment** denoted by \overline{AB} . We call A the **left endpoint** and B the **right endpoint**. The distance between the endpoints is known as the **length** of the line segment and will be denoted by AB. Two line segments with the same length are said to be **congruent**.

Any three or more points that belong to the same line are called **collinear**. See Figure 37.2



Figure 37.2

Three noncollinear points (also known as **coplanar points**) determine a **plane**, which is yet another undefined term used to describe a flat space such as a tabletop.

Subsets of a plane are called **plane shapes** or **planes figures**. We have already discussed a geometric figure, namely, a line. Another important example of a geometric figure is the concept of an angle.

By an **angle** we mean the opening between two line segments that have a common endpoint, known as the **vertex**, as shown in Figure 37.3(a). The line segments are called the **sides** of the angle.

If one of the line segments of an angle is horizontal and the other is vertical then we call the angle a **right angle**. See Figure 37.3(b). Note that the sides of an angle partition the plane into two regions, the **interior** and the **exterior** of the angle as shown in Figure 37.3(c). Two angles with the same opening are said to be **congruent**.



Figure 37.3

The Early Stages of Learning Geometry

The first stage of a child's learning geometry consists on **recognizing** geometric shapes by their appearances without paying attention to their component parts (such as the sides and the angles). For example, a rectangle may be recognized because it "looks like a door," not because it has four straight sides and four right angles. The second stage, known as **description**, students are able to describe the component parts and properties of a shape, such as how many sides it has and whether it has some congruent sides or angles. At the third stage, students become aware of **relationships** between different shapes such as a rhombus is a quadrilateral with four congruent sides and a parallelogram is a quadrilateral with parallel opposite sides, etc.

Triangles

A **triangle** is a closed figure composed exactly of three line segments called the **sides**. The points of intersection of any two line segments is called a **vertex**. Thus, a triangle has three vertices. Aslo, a triangle has three interior angles. See Figure 37.4(a).

Triangles may be classified according to their angles and sides. If exactly one

of the angle is a right angle then the triangle is called a **right triangle**. See Figure 37.4(b). A triangle with three congruent sides is called an **equilateral triangle**. See Figure 37.4(c). A triangle with two or more congruent sides is called an **isosceles** triangle. A triangle with no congruent sides is called a **scalene** triangle.



Figure 37.4

Quadrilaterals

By a **quadrilateral** we mean a closed figure with exactly four line segments (or sides). Quadrilaterals are classified as follows:

• A **trapezoid** is a quadrilateral that has exactly one pair of parallel sides. Model: the middle part of a bike frame.

• An **isosceles trapezoid** is a quadrilateral with exactly two parallel sides and the remaining two sides are congruent. Model: A water glass silhouette.

• A **parallelogram** is a quadrilateral in which each pair of opposite sides is parallel.

• A **rhombus** is a parallelogram that has four congruent sides. Model: diamond.

• A **kite** is a quadrilateral with two nonoverlapping pairs of adjacent sides that are the same length. Model: a kite.

- A rectangle is a parallelogram that has four right angles. Model: a door.
- A square is a rectangle that has four congruent sides. Model: Floor tile.



Figure 37.5

Practice Problems

Problem 37.1

Find three objects in your classroom with surfaces that suggests common geometric figures.

Problem 37.2

A fifth grader says a square is not a rectangle because a square has four congruent sides and rectangles don't have that. A second fifth grader says a square is a type of rectangle because it is a parallelogram and it has four right angles.

(a) Which child is right?

(b) How can you use the definitions to help the other child understand?

Problem 37.3

Suppose P={parallelograms}, Rh={rhombus}, S={squares}, Re={rectangles}, T={trapezoids}, and Q={quadrilaterals}. Find (a) $Rh \cap Re$ (b) $T \cap P$

Problem 37.4

Organize the sets P, Rh, S, Re, T, and Q using Venn diagram.

Problem 37.5

- (a) True or false? No scalene triangle is isosceles.
- (b) What shape is the diamond in a deck of cards?

Problem 37.6

How many squares are in the following design?



Problem 37.7

Tell whether each of the following shapes must, can, or cannot have at least one right angle.

- (a) Rhombus
- (b) Square
- (c) Trapezoid
- (d) Rectangle
- (e) Parallelogram

Problem 37.8

In which of the following shapes are both pairs of opposite sides parallel?

- (a) Rhombus
- (b) Square
- (c) Trapezoid
- (d) Rectangle
- (e) Parallelogram

Problem 37.9

A square is also which of the following?

- (a) Quadrilateral
- (b) Parallelogram
- (c) Rhombus
- (d) Rectangle

Problem 37.10

Fill in the blank with "All", "Some", or "No"

- (a) <u>rectangles are squares.</u>
- (b) _____ parallelograms are trapezoids.
 (c) _____ rhombuses are quadrilaterals.

Problem 37.11

How many triangles are in the following design?



Problem 37.12

How many squares are found in the following figure?



Problem 37.13

Given are a variety of triangles. Sides with the same length are indicated. Right angles are indicated.



- (a) Name the triangles that are scalene.
- (b) Name the triangles that are isosceles.
- (c) Name the triangles that are equilateral.
- (d) Name the triangles that contain a right angle.

Problem 37.14

(a) How many triangles are in the figure?

- (b) How many parallelograms are in the figure?
- (c) How many trapezoids are in the figure?



Problem 37.15

If possible, sketch two parallelograms that intersect at exactly

- (a) one point
- (b) two points
- (c) three points
- (d) four points.

Problem 37.16

If possible, draw a triangle and a quadrilateral that intersect at exactly

- (a) one point
- (b) two points
- (c) three points.

Problem 37.17

 $\label{eq:suppose} Suppose P=\{parallelograms\}, S=\{squares\}, T=\{trapezoids\}, and Q=\{quadrilaterals\}. Find$

(a) $P \cap S$ (b) $P \cup Q$

Problem 37.18

A fifth grader does not think that a rectangle is a type of parallelogram. Tell why it is.

Problem 37.19

Tell whether each definition has sufficient information. If it is not sufficient, tell what information is missing.

(a) A rhombus is a quadrilateral with both pairs of opposite sides parallel.

(b) A square is a quadrilateral with four congruent sides.

(c) A rhombus is a quadrilateral that has four congruent sides.

Problem 37.20

Name properties that a square, parallelogram, and rhombus have in common.

Problem 37.21

How many different line segments are contained in the following portion of a line?

