1 Pólya's Problem-Solving Process

Problem-solving is the cornerstone of school mathematics. The main reason of learning mathematics is to be able to solve problems. Mathematics is a powerful tool that can be used to solve a vast variety of problems in technology, science, business and finance, medecine, and daily life.

It is strongly believed that the most efficient way for learning mathematical concepts is through problem solving. This is why the National Council of Teachers of Mathematics NCTM advocates in *Principles and Standards for School Mathematics,* published in 2000, that mathematics instruction in American schools should emphasize on problem solving and quantitative reasoning. So, the conviction is that children need to learn to think about quantitative situations in insightful and imaginative ways, and that mere memorization of rules for computation is largely unproductive.

Of course, if children are to learn problem solving, their teachers must themselves be competent problem solvers and teachers of problem solving. The techniques discussed in this and the coming sections should help you to become a better problem solver and should show you how to help others develop their problem-solving skills.

Pólya's Four-Step Process

In his book *How to Solve It*, George Pólya identifies a four-step process that forms the basis of any serious attempt at problem solving. These steps are:

Step 1. Understand the Problem

Obviously if you don't understand a problem, you won't be able to solve it. So it is important to understand what the problem is asking. This requires that you read slowly the problem and carefully understand the information given in the problem. In some cases, drawing a picture or a diagram can help you understand the problem.

Step 2. Devise a Plan

There are many different types of plans for solving problems. In devising a plan, think about what information you know, what information you are looking for, and how to relate these pieces of information. The following are few common types of plans:

• Guess and test: make a guess and try it out. Use the results of your guess to guide you.

- Use a variable, such as x.
- Draw a diagram or a picture.
- Look for a pattern.
- Solve a simpler problem or problems first- this may help you see a pattern you can use.
- make a list or a table.

Step 3. Carry Out the Plan

This step is considered to be the hardest step. If you get stuck, modify your plan or try a new plan. Monitor your own progress: if you are stuck, is it because you haven't tried hard enough to make your plan work, or is it time to try a new plan? Don't give up too soon. Students sometimes think that they can only solve a problem if they've seen one just like it before, but this is not true. Your common sense and natural thinking abilities are powerful tools that will serve you well if you use them. So don't underestimate them!

Step 4. Look Back

This step helps in identifying mistakes, if any. Check see if your answer is plausible. For example, if the problem was to find the height of a telephone pole, then answers such as 2.3 feet or 513 yards are unlikely-it would be wise to look for a mistake somewhere. *Looking back* also gives you an opportunity to make connections: Have you seen this type of answer before? What did you learn from this problem? Could you use these ideas in some other way? Is there another way to solve the problem? Thus, when you *look back*, you have an opportunity to learn from your own work.

Solving Applied Problems

The term "word problem" has only negative connotations. It's better to think of them as "applied problems." These problems should be the most interesting ones to solve. Sometimes the "applied" problems don't appear very realistic, but that's usually because the corresponding real applied problems are too hard or complicated to solve at your current level. But at least you get an idea of how the math you are learning can help solve actual real-world problems.

Many problems in this book will be word problems. To solve such problems, one translates the words into an equivalent problem using mathematical symbols, solves this equivalent problem, and then interprets the answer. This process is summarized in Figure 1.1

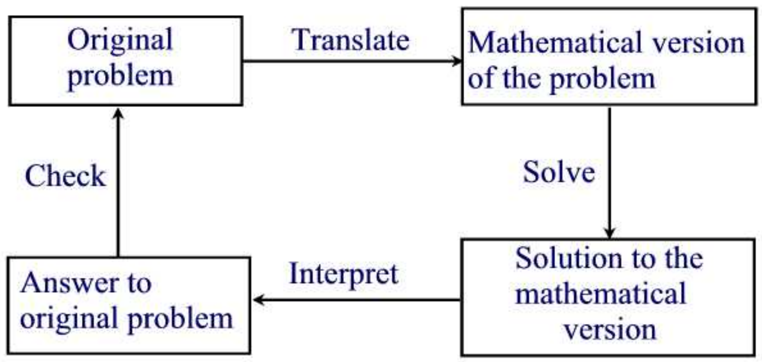

Figure 11

Example 1.1
In each of the following situations write the equation that describes the situation. Do not solve the equation:

(a) Herman's selling house is x dollars. The real estate agent received 7% of the selling price, and Herman's received \$84,532. What is the selling price of the house?
(b) The sum of three consecutive integers is 48. Find the integers.

Solution.
(a) The equation describing this situation is

$$x - 0.07x = 84,532.$$

(b) If x is the first integer then $x + 1$ and $x + 2$ are the remaining integers. Thus,

$$x + (x + 1) + (x + 2) = 48. \blacksquare$$

Practice Problems

In each of the following problems write the equation that describes each situation. Do not solve the equation.

Problem 1.1
Two numbers differ by 5 and have a product of 8. What are the two numbers?

Problem 1.2
Jeremy paid for his breakfest with 36 coins consisting of nickels and dimes. If the bill was \$3.50, then how many of each type of coin did he use?

Problem 1.3
The sum of three consecutive odd integers is 27. Find the three integers.

Problem 1.4
At an 8% sales tax rate, the sales tax Peter's new Ford Taurus was \$1,200. What was the price of the car?

Problem 1.5
After getting a 20% discount, Robert paid \$320 for a Pioneer CD player for his car. What was the original price of the CD?

Problem 1.6
The length of a rectangular piece of property is 1 foot less than twice the width. The perimeter of the property is 748 feet. Find the length and the width.

Problem 1.7
Sarah is selling her house through a real estate agent whose commission rate is 7%. What should the selling price be so that Sarah can get the \$83,700 she needs to pay off the mortgage?

Problem 1.8
Ralph got a 12% discount when he bought his new 1999 Corvette Coupe. If the amount of his discount was \$4,584, then what was the original price?

Problem 1.9
Julia framed an oil painting that her uncle gave her. The painting was 4 inches longer than it was wide, and it took 176 inches of frame molding. What were the dimensions of the picture?

Problem 1.10
If the perimeter of a tennis court is 228 feet and the length is 6 feet longer than twice the width, then what are the length and the width?