1. (a) True; each of the first two lines has a direction vector parallel to the direction vector of the third line, so these vectors are each scalar multiples of the third direction vector. Then the first two direction vectors are also scalar multiples of each other, so these vectors, and hence the two lines, are parallel.

(b) False; for example, the $x$- and $y$-axes are both perpendicular to the $z$-axis, yet the $x$- and $y$-axes are not parallel.

(c) True; each of the first two planes has a normal vector parallel to the normal vector of the third plane, so these two normal vectors are parallel to each other and the planes are parallel.

(d) False; for example, the $xy$- and $yz$-planes are not parallel, yet they are both perpendicular to the $xz$-plane.

(e) False; the $x$- and $y$-axes are not parallel, yet they are both parallel to the plane $z = 1$.

(f) True; if each line is perpendicular to a plane, then the lines’ direction vectors are both parallel to a normal vector for the plane. Thus, the direction vectors are parallel to each other and the lines are parallel.

(g) False; the planes $y = 1$ and $z = 1$ are not parallel, yet they are both parallel to the $x$-axis.

(h) True; if each plane is perpendicular to a line, then any normal vector for each plane is parallel to a direction vector for the line. Thus, the normal vectors are parallel to each other and the planes are parallel.

(i) True; see Figure 9 and the accompanying discussion.

(j) False; they can be skew, as in Example 3.

(k) True. Consider any normal vector for the plane and any direction vector for the line. If the normal vector is perpendicular to the direction vector, the line and plane are parallel. Otherwise, the vectors meet at an angle $\theta$, $0^\circ \leq \theta < 90^\circ$, and the line will intersect the plane at an angle $90^\circ - \theta$.

3. For this line, we have $\mathbf{r}_0 = 2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, so a vector equation is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v} = (2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = (2 + 3t)\mathbf{i} + (2.4 + 2t)\mathbf{j} + (3.5 - t)\mathbf{k}$ and parametric equations are $x = 2 + 3t$, $y = 2.4 + 2t$, $z = 3.5 - t$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n} = (1, 3, 1)$. So $\mathbf{r}_0 = \mathbf{i} + 6\mathbf{k}$, and we can take $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. Then a vector equation is $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k}) = (1 + t)\mathbf{i} + 3t\mathbf{j} + (6 + t)\mathbf{k}$, and parametric equations are $x = 1 + t$, $y = 3t$, $z = 6 + t$. 

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7. The vector \( \mathbf{v} = \langle 2 - 0, 1 - \frac{1}{2}, -3 - 1 \rangle = \langle 2, \frac{1}{2}, -4 \rangle \) is parallel to the line. Letting \( P_0 = (2, 1, -3) \), parametric equations are \( x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t \), while symmetric equations are \( \frac{x - 2}{2} = \frac{y - 1}{1/2} = \frac{z + 3}{-4} \) or \( \frac{x - 2}{2} = 2y - 2 = \frac{z + 3}{-4} \).

9. The line has direction \( \mathbf{v} = \langle 1, 2, 1 \rangle \). Letting \( P_0 = (1, -1, 1) \), parametric equations are \( x = 1 + t, y = -1 + 2t, z = 1 + t \) and symmetric equations are \( x - 1 = \frac{y + 1}{2} = z - 1 \).

11. Direction vectors of the lines are \( \mathbf{v}_1 = \langle -2 - (-4), 0 - (-6), -3 - 1 \rangle = \langle 2, 6, -4 \rangle \) and \( \mathbf{v}_2 = \langle 5 - 10, 3 - 18, 14 - 4 \rangle = \langle -5, -15, 10 \rangle \), and since \( \mathbf{v}_2 = -\frac{5}{2} \mathbf{v}_1 \), the direction vectors and thus the lines are parallel.

13. (a) The line passes through the point \( (1, -5, 6) \) and a direction vector for the line is \( \langle -1, 2, -3 \rangle \), so symmetric equations for the line are \( \frac{x - 1}{-1} = \frac{y + 5}{2} = \frac{z - 6}{-3} \).

(b) The line intersects the \( xy \)-plane when \( z = 0 \), so we need \( \frac{x - 1}{-1} = \frac{y + 5}{2} = \frac{0 - 6}{-3} \) or \( \frac{x - 1}{-1} = 2 \) \( \Rightarrow \) \( x = -1 \), \( \frac{y + 5}{2} = 2 \) \( \Rightarrow \) \( y = -1 \). Thus the point of intersection with the \( xy \)-plane is \( (1, -1, 0) \). Similarly for the \( yz \)-plane, we need \( x = 0 \) \( \Rightarrow \) \( 1 = \frac{y + 5}{2} = \frac{z - 6}{-3} \) \( \Rightarrow \) \( y = -3, z = 3 \). Thus the line intersects the \( yz \)-plane at \( (0, -3, 3) \). For the \( xz \)-plane, we need \( y = 0 \) \( \Rightarrow \) \( \frac{x - 1}{-1} = \frac{5}{2} = \frac{z - 6}{-3} \) \( \Rightarrow \) \( x = -\frac{3}{2}, z = -\frac{3}{2} \). So the line intersects the \( xz \)-plane at \( (-\frac{3}{2}, 0, -\frac{3}{2}) \).

15. From , the line segment from \( \mathbf{r}_0 = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} \) to \( \mathbf{r}_1 = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} \) is \( \mathbf{r}(t) = (1 - t)\mathbf{r}_0 + t\mathbf{r}_1 = (1 - t)(2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(4\mathbf{i} + 6\mathbf{j} + \mathbf{k}) = (2 - t)\mathbf{i} + (6t - 7)\mathbf{j} + (4 - 3t)\mathbf{k}, 0 \leq t \leq 1 \).

17. Since the direction vectors \( \langle 2, -1, 3 \rangle \) and \( \langle 4, -2, 5 \rangle \) are not scalar multiples of each other, the lines are not parallel. For the lines to intersect, we must be able to find one value of \( t \) and one value of \( s \) that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations: \( 3 + 2t = 1 + 4s, 4 - t = 3 - 2s, 1 + 3t = 4 + 5s \). Solving the last two equations we get \( t = 1, s = 0 \) and checking, we see that these values don’t satisfy the first equation. Thus the lines aren’t parallel and don’t intersect, so they must be skew lines.
19. Since the direction vectors \( \langle 1, -2, -3 \rangle \) and \( \langle 1, 3, -7 \rangle \) aren't scalar multiples of each other, the lines aren't parallel. Parametric equations of the lines are \( L_1: x = 2 + t, y = 3 - 2t, z = 1 - 3t \) and \( L_2: x = 3 + s, y = -4 + 3s, z = 2 - 7s \). Thus, for the lines to intersect, the three equations \( 2 + t = 3 + s, 3 - 2t = -4 + 3s, \) and \( 1 - 3t = 2 - 7s \) must be satisfied simultaneously. Solving the first two equations gives \( t = 2, s = 1 \) and checking, we see that these values do satisfy the third equation, so the lines intersect when \( t = 2 \) and \( s = 1 \), that is, at the point \( (4, -1, -5) \).

21. \( \mathbf{i} + 4 \mathbf{j} + \mathbf{k} = \langle 1, 4, 1 \rangle \) is a normal vector to the plane and \( (-1, \frac{1}{2}, 3) \) is a point on the plane, so setting \( a = 1, b = 4, c = 1, x_0 = -1, y_0 = \frac{1}{2}, z_0 = 3 \) in Equation 7 gives \( 1[x - (-1)] + 4 \left( y - \frac{1}{2} \right) + 1(z - 3) = 0 \) or \( x + 4y + z = 4 \) as an equation of the plane.

23. Since the two planes are parallel, they will have the same normal vectors. So we can take \( \mathbf{n} = \langle 5, -1, -1 \rangle \), and an equation of the plane is \( 5(x - 1) - 1(y - (-1)) - 1(z - (-1)) = 0 \) or \( 5x - y - z = 7 \).

29. A direction vector for the line of intersection is \( \mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, -1 \rangle \times \langle 2, -1, 3 \rangle = \langle 2, -5, -3 \rangle \), and \( \mathbf{a} \) is parallel to the desired plane. Another vector parallel to the plane is the vector connecting any point on the line of intersection to the given point \( (-1, 2, 1) \) in the plane. Setting \( x = 0 \), the equations of the planes reduce to \( y - z = 2 \) and \( -y + 3z = 1 \) with simultaneous solution \( y = \frac{7}{2} \) and \( z = \frac{3}{2} \). So a point on the line is \( (0, \frac{7}{2}, \frac{3}{2}) \) and another vector parallel to the plane is \( \langle -1, -\frac{3}{2}, -\frac{1}{2} \rangle \). Then a normal vector to the plane is \( \mathbf{n} = \langle 2, -5, -3 \rangle \times \langle -1, -\frac{3}{2}, -\frac{1}{2} \rangle = \langle -2, 4, -8 \rangle \) and an equation of the plane is \( -2(x + 1) + 4(y - 2) - 8(z - 1) = 0 \) or \( x - 2y + 4z = -1 \).

31. If a plane is perpendicular to two other planes, its normal vector is perpendicular to the normal vectors of the other two planes. Thus \( \langle 2, 1, -2 \rangle \times \langle 1, 0, 3 \rangle = \langle 3 - 0, -2 - 6, 0 - 1 \rangle = \langle 3, -8, -1 \rangle \) is a normal vector to the desired plane. The point \( (1, 5, 1) \) lies on the plane, so an equation is \( 3(x - 1) - 8(y - 5) - (z - 1) = 0 \) or \( 3x - 8y - z = -38 \).

33. Substitute the parametric equations of the line into the equation of the plane: \( (3 - t) - (2 + t) + 2(5t) = 9 \) \( \Rightarrow \) \( 8t = 8 \) \( \Rightarrow \) \( t = 1 \). Therefore, the point of intersection of the line and the plane is given by \( x = 3 - 1 = 2, y = 2 + 1 = 3, \) and \( z = 5(1) = 5 \), that is, the point \( (2, 3, 5) \).
35. Normal vectors for the planes are \( \mathbf{n}_1 = \langle 1, 1, 1 \rangle \) and \( \mathbf{n}_2 = \langle 1, -1, 1 \rangle \). The normals are not parallel, so neither are the planes. Furthermore, \( \mathbf{n}_1 \cdot \mathbf{n}_2 = 1 - 1 + 1 = 1 \neq 0 \), so the planes aren’t perpendicular. The angle between them is given by
\[
\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| \cdot |\mathbf{n}_2|} = \frac{1}{\sqrt{3} \sqrt{3}} = \frac{1}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5^\circ.
\]

37. The normals are \( \mathbf{n}_1 = \langle 1, -4, 2 \rangle \) and \( \mathbf{n}_2 = \langle 2, -8, 4 \rangle \). Since \( \mathbf{n}_2 = 2\mathbf{n}_1 \), the normals (and thus the planes) are parallel.

39. (a) To find a point on the line of intersection, set one of the variables equal to a constant, say \( z = 0 \). (This will fail if the line of intersection does not cross the \( xy \)-plane; in that case, try setting \( x \) or \( y \) equal to 0.) The equations of the two planes reduce to \( x + y = 1 \) and \( x + 2y = 1 \). Solving these two equations gives \( x = 1, y = 0 \). Thus a point on the line is \( (1, 0, 0) \).

A vector \( \mathbf{v} \) in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so we can take \( \mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 2, 2 \rangle = \langle 2 - 1, 1 - 2, 2 - 1 \rangle = \langle 0, -1, 1 \rangle \). By Equations 2, parametric equations for the line are \( x = 1, y = -t, z = t \).

(b) The angle between the planes satisfies \( \cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| \cdot |\mathbf{n}_2|} = \frac{1 + 2 + 2}{\sqrt{3} \sqrt{9}} = \frac{5}{3 \sqrt{3}} \). Therefore \( \theta = \cos^{-1}\left(\frac{5}{3 \sqrt{3}}\right) \approx 15.8^\circ \).

49. By Equation 9, the distance is
\[
D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{|18|}{\sqrt{49}} = \frac{18}{7}.
\]

51. Put \( y = z = 0 \) in the equation of the first plane to get the point \( (2, 0, 0) \) on the plane. Because the planes are parallel, the distance \( D \) between them is the distance from \( (2, 0, 0) \) to the second plane. By Equation 9,
\[
D = \frac{|4(2) - 6(0) + 2(0) - 3|}{\sqrt{4^2 + (-6)^2 + 2^2}} = \frac{5}{\sqrt{56}} = \frac{5}{2 \sqrt{14}} = \frac{5 \sqrt{14}}{28}.
\]