Describe in words how the graph of each function below would differ from the graph of \( f(x) \).

1. \( f(x - 7) \)  
2. \( -f(x) \)  
3. \( f(x) - 5 \)  
4. \( -f(-x) \)  
5. \( 7f(x) \)  
6. \( f(x + 2) + 9 \)  
7. \( f(-x) \)  
8. \( \frac{3}{2} f(x) \)

9. Graph each of the following functions and describe the transformations that have been applied to the basic function \( f(x) = x^2 \).
   a) \( f(x) = x^2 + 2 \)  
   b) \( f(x) = (x - 3)^2 - 5 \)  
   c) \( f(x) = -\frac{1}{2} x^2 \)

10. Graph each of the following functions and describe the transformations that have been applied to the basic function \( f(x) = \sqrt{x} \).
    a) \( f(x) = \sqrt{x} + 4 \)  
    b) \( f(x) = 3\sqrt{x} \)  
    c) \( f(x) = \sqrt{-x} \)

11. Write equations for each of the following and sketch their graphs.
    a) The graph of \( y = x^3 \) is shifted right 4 units and down 2 units.
    b) The graph of \( y = \sqrt{x} \) is reflected across the \( y \)-axis and shifted up 3 units.
    c) The graph of \( y = |x| \) is reflected across the \( x \)-axis and vertically stretched by a factor of 4.

12. Determine whether each function is even, odd, or neither, and state whether its graph is symmetric with respect to the \( x \)-axis, the \( y \)-axis, the origin, or none of these.
    a) \( f(x) = 2x + 20 \)  
    b) \( f(x) = 3x^4 - 1 \)  
    c) \( f(x) = 4x^3 - 12x \)
13. Given \( f(x) = x^2 - 3 \) and \( g(x) = 4x + 1 \), find the following, in simplified form.

a) \((f + g)(x)\)  
b) \((f - g)(x)\)  
c) \((f \cdot g)(x)\)  
d) \(\left(\frac{f}{g}\right)(x)\)  
e) \((f \circ g)(x)\)  
f) \((g \circ f)(x)\)  
g) \((f + g)(5)\)  
h) \((f \cdot g)(-2)\)  
i) \((f \circ g)(0)\)

j) State the domain of \((f \cdot g)(x)\).  
k) State the domain of \(\left(\frac{f}{g}\right)(x)\).

14. Given \( f(x) = \sqrt{x - 2} \) and \( g(x) = x^2 \), find the following, in simplified form.

a) \((f + g)(x)\)  
b) \((f - g)(x)\)  
c) \((f \cdot g)(x)\)  
d) \(\left(\frac{f}{g}\right)(x)\)  
e) \((f \circ g)(x)\)  
f) \((g \circ f)(x)\)  
g) \((f - g)(6)\)  
h) \(\left(\frac{f}{g}\right)(11)\)  
i) \((g \circ f)(3)\)

j) State the domain of \((g \circ f)(x)\).  
k) State the domain of \(\left(\frac{g}{f}\right)(x)\).

15. Given \( f(x) = 2x - 3 \) and \( g(x) = \sqrt{x + 5} \), find the following function compositions.

a) \((f \circ g)(x)\)  
b) \((g \circ f)(x)\)  
c) \((f \circ g)(4)\)  
d) \((g \circ f)(-1)\)  
e) State the domain of \((f \circ g)(x)\).  
f) State the domain of \((g \circ f)(x)\).
16. Suppose the weekly cost for the production and sale of bicycles is $C(x) = 23x + 3420$ dollars and the total revenue is given by $R(x) = 89x$ dollars, where $x$ is the number of bicycles.

a) Write the equation of the function that models weekly profit from the production and sale of $x$ bicycles.

b) What is the profit on the production and sale of 150 bicycles?

17. Determine whether or not each of the following functions is one-to-one.

a) $\{(2,5),(3,2),(4,5)\}$  
b) $\{(1,3),(2,2),(3,4)\}$  
c) $f(x) = 3x^2 + 1$

d) $y = 3x - 4$  
e) $f(x) = 4x^3$  
f) $f(x) = \sqrt{x - 5}$

18. Find the inverse, $f^{-1}(x)$, for each of the given functions, and then sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same set of axes.

a) $f(x) = \sqrt{x + 3}$  
b) $f(x) = 2x^3 - 4$  
c) $f(x) = 3x + 9$

19. Find the inverse, $f^{-1}(x)$, for each of the given functions, and state the domain and range of both $f(x)$ and $f^{-1}(x)$.

a) $f(x) = \frac{4}{x-3}$  
b) $f(x) = \sqrt{5x + 4}$

20. Use compositions to determine whether each pair of functions defined as follows are inverses of each other.

a) $f(x) = \frac{1}{3}x - 1$; $g(x) = 5x + 5$  
b) $f(x) = \sqrt{2x + 3}$; $g(x) = \frac{x^3 - 3}{2}$

Solve each of the following equations algebraically and check your solutions.

21. $\sqrt[3]{x-1} = -2$  
22. $\sqrt[3]{3x-2} + 2 = x$  
23. $\sqrt[4]{4x+5} = \sqrt{x^2-7}$
24. $|2x - 5| = 3$

25. $|x^2 - 5x| = 6$

26. $|3x - 1| = 4x$

For each of the polynomial functions given in #30-32, do the following:

a) Describe the end behavior of the graph.
b) Determine the maximum number of turning points the graph could have.
c) Graph the function on a window which shows a complete graph. (Sketch your graph.)
d) Find the coordinates of the local minimum point(s).
e) Find the coordinates of the local maximum point(s).

27. $y = x^3 + 4x^2 + 5$

28. $y = -0.25x^4 - x^3 + 2x^2 + 6x + 3$

29. $y = 9x - x^3$

For each of the polynomial functions given in #33-35, do the following:

a) Describe the end behavior of the graph.
b) Determine the maximum number of $x$-intercepts the graph could have.
c) Graph the function on a window which shows a complete graph. (Sketch your graph.)
d) Determine all of the $x$-intercepts.

30. $y = -x^4 + 3x^3 + 8x^2 - 12x - 16$

31. $y = x^3 - 4x$

32. $y = x^4 - 10x^2 + 9$

33. The percent of the U.S. population that was foreign born during the years 1900-2000 is given by the polynomial function $y = 0.0000591x^3 - 0.00675x^2 + 0.0523x + 14.147$, where $x$ is the number of years after 1900.

a) Describe the end behavior of the graph.

b) What window should be used to show the entire time period 1900-2000 and to show all possible percents?

c) Graph the function on the appropriate window and determine the year that the percentage of the foreign-born population was a minimum. (Sketch your graph.)

d) What percent of the population was foreign-born in the year 1910? In 1990?
34. A firm has total weekly revenue (in dollars) for its product given by
\[ R(x) = 2000x + 30x^2 - 0.3x^3, \] where \( x \) is the number of units sold.

a) Describe the end behavior of the graph.

b) Graph the function on the window \([0, 150] \times [0, 220,000]\). (Sketch the graph.) Is this a complete graph?

c) Use the graph to determine the number of units which must be sold to produce the maximum revenue, and determine the maximum revenue that can be made.

35. Solve each equation by factoring.

a) \( 9x^3 - 3x^2 = 0 \)

b) \( x^3 + 5x^2 - 9x - 45 = 0 \)

c) \( 120x^2 - 20x^3 = 0 \)

36. Solve each equation by the root method.

a) \( 5x^2 - 20 = 0 \)

b) \( 4x^3 - 20 = 0 \)

c) \( \frac{1}{2}x^3 - 4 = 0 \)

37. Use factoring and the root method to solve each polynomial equation.

a) \( 3x^4 - 18x^2 = 0 \)

b) \( \frac{1}{2}x^3 - \frac{25}{2}x = 0 \)

c) \( 1000x - 0.1x^3 = 0 \)

38. Given the zeros of the polynomial \( P(x) \), give two possible equations for \( P(x) \), each with different end behavior.

a) \(-3, 0, 2\)

b) \(-5, -1, 1, 3\)
39. The revenue from the sale of a product is given by the function \( R = 400x - x^3 \), where \( x \) is the number of units sold. Determine the number of units which must be sold to give zero revenue.

40. Give the solutions of each equation based on the graphs of \( p(x) \) shown below. Then write a possible equation for each function. Assume that the scale on the x-axis is 1, but do not make the same assumption for the y-axis.

a) \( p(x) = 0 \)

b) \( p(x) = 0 \)