6.1 The Law of Sines

Everything we have done so far has relied on right triangle trigonometry, in which we have one right angle, a hypotenuse and two legs. However, not all triangles are right triangles, so to handle **oblique triangles** (triangles which do not contain right angles), we have two new tools: the Law of Sines and the Law of Cosines. In this section we will look at the **Law of Sines**.

Given a triangle with angles A, B, and C, and sides opposite them a, b, and c, respectively, as shown below, the following relationships hold:

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Note that since we are no longer dealing with a right triangle, there is no hypotenuse, and thus all the trig ratios are null and void!

The Law of Sines: \[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

The law of sines requires that you have at least one corresponding pair (angle and side opposite it) and one additional side or angle.

Examples: Given the triangles below, use the Law of Sines to find the missing sides and angles. Round side lengths to the nearest tenth and angles to the nearest degree.

1) \[
\begin{align*}
8.5 & \quad b \\
70^\circ & \quad 62^\circ \\
9 & \\
A & \\
C
\end{align*}
\]

\[
\frac{9}{\sin 70^\circ} = \frac{b}{\sin 62^\circ} \Rightarrow b \approx 8.5
\]

\[
\frac{9}{\sin 70^\circ} = \frac{c}{\sin 48^\circ} \Rightarrow c \approx 7.1
\]

2) \[
\begin{align*}
8 & \quad 7 \\
32^\circ & \quad 37^\circ \\
A & \\
C
\end{align*}
\]

\[
\frac{7}{\sin 32^\circ} = \frac{8}{\sin B} \Rightarrow B \approx 37^\circ
\]

\[
\frac{7}{\sin 32^\circ} = \frac{c}{\sin 37^\circ} \Rightarrow c \approx 12.3
\]

\[
\sin \theta \approx \sin(32^\circ) \approx 0.525
\]

\[
37.27361499
\]
Construct an oblique triangle with the given information and then use the Law of Sines to find the missing values. Round side lengths to the nearest tenth and angles to the nearest degree.

1) $a = 20$, $b = 14$, $A = 40^\circ$
2) $B = 5^\circ$, $C = 125^\circ$, $b = 20$
3) $a = 10$, $B = 50^\circ$, $C = 70^\circ$
4) $a = 5$, $c = 7$, $A = 42^\circ$
The area of an oblique triangle can be found by using the following formula:

\[
\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B
\]

Find the area of each triangle described below.

1) \( A = 36^\circ, b = 3 \text{ feet}, c = 5 \text{ feet} \)

\[
\text{Area} = \frac{1}{2} \cdot 3 \cdot 5 \cdot \sin 36^\circ \approx 4.4 \text{ sq ft}
\]

2) \( C = 100^\circ, a = 12 \text{ inches}, b = 10 \text{ inches} \)

\[
\text{Area} = \frac{1}{2} \cdot 12 \cdot 10 \cdot \sin 100
\]

\[
= 59.1 \text{ square inches}
\]

Applications:

1) Two fire-lookout stations are 12 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N32°E (32° east of north), and the bearing of the fire from station B is N49°W (49° west of north). How far, to the nearest tenth of a mile, is the fire from each lookout station?

\[
\frac{a \cdot \sin 58^\circ}{\sin 81^\circ} = \frac{12 \cdot \sin 58^\circ}{\sin 81^\circ}
\]

\[
a \approx 10.3 \text{ miles from } B
\]

\[
\frac{b \cdot \sin 41^\circ}{\sin 81^\circ} = \frac{12 \cdot \sin 41^\circ}{\sin 81^\circ}
\]

\[
b \approx 8.0 \text{ miles from } A
\]

2) A surveyor needs to determine the distance between two points that lie on opposite banks of a river. The figure below shows that 300 yards are measured along one bank. The angles from each end of this line segment to a point on the opposite bank are 62° and 53°. Find the distance between A and B, to the nearest tenth of a yard.

\[
\frac{c \cdot \sin 53^\circ}{\sin 65^\circ} = \frac{300}{\sin 65^\circ}
\]

\[
c \approx 264.4 \text{ yds.}
\]