1.6: Transformations of Functions

In this section we will examine the graphs of some common algebraic functions and observe the results of several different transformations on these and other graphs.

Table 1.3 on p. 216 shows several common graphs with which you should be or become familiar.
**Vertical shifts:** If $c$ is a positive real number, the graph of $f(x) + c$ is the graph of $f(x)$ shifted up $c$ units, and the graph of $f(x) - c$ is the graph of $f(x)$ shifted down $c$ units.

Compare the graphs of the following three functions.

\[ f(x) = x^2, \quad f(x) + 3 = x^2 + 3, \quad f(x) - 3 = x^2 - 3 \]
**Horizontal shifts**: If $c$ is a positive real number, the graph of $f(x + c)$ is the graph of $f(x)$ shifted left $c$ units, and the graph of $f(x - c)$ is the graph of $f(x)$ shifted right $c$ units.

Compare the graphs of the following three functions.

$$f(x) = \sqrt{x}, \quad f(x + 3) = \sqrt{x + 3}, \quad f(x - 3) = \sqrt{x - 3}$$

$$(3,0)$$
**Vertical Stretches/Compressions:** Suppose $c > 0$.

- $cf(x)$ is a **vertical stretch** of $f(x)$ if $c > 1$.
- $cf(x)$ is a **vertical compression** of $f(x)$ if $c < 1$.

Compare the graphs of

- $f(x) = x - 2$
- $2 \cdot f(x) = 2(x - 2)$
- $\frac{1}{2} f(x) = \frac{1}{2} (x - 2)$
Horizontal Stretches/Compressions: Suppose $c > 0$.

$f(cx)$ is a horizontal compression of $f(x)$ if $c > 1$.

$f(cx)$ is a horizontal stretch of $f(x)$ if $c < 1$.

Compare the graphs of

\[
\begin{align*}
    f(x) &= x - 2 \\
    f(2x) &= 2x - 2 \\
    f\left(\frac{1}{2}x\right) &= \frac{1}{2}x - 2
\end{align*}
\]
Reflections

-f(x) is a reflection of f(x) across the x-axis.

f(-x) is a reflection of f(x) across the y-axis.

-f(-x) is a reflection of f(x) across the origin.

Compare the graphs of \( f(x) = \sqrt{x} \)

\[ -f(x) = -\sqrt{x} \]
\[ f(-x) = \sqrt{-x} \]
\[ f(-x) = -\sqrt{-x} \]
Examples:
1) Given the graph of \( f(x) \), sketch the graph of each transformation.

- \( f(x) \)
- \( f(x) - 3 \)
- \( f(x - 2) + 1 \)
- \(-f(x)\)

2) Given the graph of \( f(x) \), sketch the graph of each transformation.

- \( f(x) \)
- \( 2f(x) \)
- \( f(\frac{1}{2}x) \)
- \( f(-x) \)
Describe how the graph of each given function would differ from the graph of \( f(x) \).

3) \( f(2x) - 5 \)  
   horizontally compressed + shifted down 5

4) \( f(-x) + 2 \)  
   reflected across y-axis + shifted up 2

5) \( \frac{1}{2} f(x + 7) \)  
   reflected across x-axis, vertically compressed + shifted right 7

Functions \( f \) and \( g \) are graphed on the same grid. If \( g \) is obtained from \( f \) through a sequence of transformations, find an equation for \( g \).

6)

7)  
\[ f(x) = x^2 \]  
\[ g(x) = -(x+3)^2 - 1 \]
Graph the given transformations of the following basic functions:

\[ f(x) = |x|, \quad f(x) = x^2, \quad f(x) = x^3, \quad f(x) = \sqrt{x}, \quad f(x) = \sqrt[3]{x} \]

1) \[ f(x) = |x - 2| + 3 \]

2) \[ f(x) = -3x^2 \]

3) \[ f(x) = \frac{1}{3}x^3 - 1 \]

4) \[ f(x) = 2\sqrt{-x} \]
5) \( f(x) = -\sqrt[3]{x + 4} \)

6) \( f(x) = (x + 3)^2 - 4 \)