Conic Sections: Parabolas, Circles, Ellipses, and Hyperbolas

The conic sections have equations which are quadratic in form. The general quadratic equation in $x$ and $y$ is as follows: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A$, $B$, $C$, $D$, $E$, and $F$ are constants. We will look at quadratic equations in which $B = 0$. If either $A$ or $C$ (but not both) is 0, we have a parabola. If $A$ and $C$ are both non-zero and have the same sign, we have an ellipse, and if they have different signs, we have a hyperbola.
Today we will focus on the parabola, so the equation will involve either an $x^2$ term or a $y^2$ term. We have discussed quadratic functions previously, in which we have equations of the form $y = a(x - h)^2 + k$. This is a parabola which opens either up or down (depending on the sign of "a") and has vertex $(h, k)$.

A parabola may also open either left or right, in which case its equation would be $x = a(y - k)^2 + h$. In this case, if "a" is positive, the parabola opens right, and if "a" is negative, the parabola opens left. The vertex is still $(h, k)$.

The geometric definition of a parabola is the set of points in a plane which are equidistant from a given point, called the **focus**, and a given line, called the **directrix**. The **axis of symmetry** is the line that goes through the focus and is perpendicular to the directrix. The point of intersection of the axis and the parabola is the **vertex**.
The distance from the vertex to the focus (as well as the distance from the vertex to the directrix) is $|c|$, where $c = \frac{1}{4a}$. The focus and the vertex both lie on the axis of symmetry, and are both ordered pairs, whereas the directrix is expressed as the equation of a line.

\[ y = 2x^2 \]

\[ x = \frac{1}{2}y^2 \]
State the vertex of each parabola below, and make a sketch showing the direction the parabola opens. Then determine the value of "c" for each.

\[ y = -(x - 5)^2 + 8 \]
\[ x = 3(y - 7)^2 - 5 \]

\[ x = -\frac{1}{4}(y + 3)^2 + 6 \]
\[ y = 8(x + 4)^2 \]
State the vertex of each parabola below, and make a sketch showing the direction the parabola opens. Then determine the value of "c" for each.

1) \( y = 2(x - 3)^2 + 5 \)  
2) \( y = -3(x + 4)^2 - 7 \)

3) \( y = \frac{1}{2}(x + 1)^2 \)  
4) \( y = -2x^2 + 3 \)

5) \( x = 4(y - 2)^2 + 5 \)  
6) \( x = -3(y + 4)^2 + 1 \)

7) \( x = (y + 5)^2 - 3 \)  
8) \( x = -y^2 + 4 \)
State the vertex of each parabola whose equation is given, and make a sketch showing the direction that the parabola opens.

1) \( y = 2(x - 3)^2 + 5 \)  
   v: (3, 5)  

2) \( y = -3(x + 4)^2 - 7 \)  
   v: (-4, -7)  

3) \( y = \frac{1}{2}(x+1)^2 \)  
   v: (-1, 0)  

4) \( y = -2x^2 + 3 \)  
   v: (0, 3)  

5) \( x = 4(y - 2)^2 + 5 \)  
   v: (5, 2)  

6) \( x = -3(y + 4)^2 + 1 \)  
   v: (1, -4)  

7) \( x = \frac{1}{4}(y + 5)^2 - 3 \)  
   (-3, -5)  

8) \( x = -y^2 + 4 \)  
   v: (0, 4)  

9) \( y = \frac{1}{12}x^2 \)  
   v: (0, 0)  

For each of the equations above, determine the value of “c”.

1) \( c = 1/8 \)  
2) \( c = -1/12 \)  
3) \( c = 1/2 \)  

4) \( c = -1/8 \)  
5) \( c = 1/16 \)  
6) \( c = -1/12 \)  

7) \( c = 1 \)  
8) \( c = -1/4 \)  
9) \( c = 3 \)
For each of the given equations, determine the vertex, the focus, and the directrix. Then graph each parabola.

\[ y = 2x^2 \]  \[ x = \frac{1}{2}y^2 \]
For each of the given equations, determine the vertex, the focus, and the directrix. Then graph each parabola.

\[ y = (x - 3)^2 + 4 \]

\[ x = -4(y + 5)^2 + 1 \]

\[ x = \frac{1}{2} (y - 1)^2 + 2 \]

\[ y = -3(x + 1)^2 - 2 \]
Write each of the given equations in vertex form and then determine the vertex, the focus, and the directrix.

\[ y = 2x^2 - 8x + 5 \quad x = -y^2 - 4y - 7 \]

\[ x = 3y^2 + 12y - 1 \quad y = \frac{1}{2}x^2 - 3x + 1 \]
We can also write the equation of a parabola if we are given certain information about its graph.

Examples: Write the equation of each parabola described below.

1) vertex: (0, 0); focus: (3, 0)  
2) vertex: (0, 0); focus: (0, 4)  
3) vertex: (3, 4); directrix: y = 2  
4) vertex: (7, 5); directrix: x = 2  
5) focus: (-2, 2); directrix: y = -2  
6) focus: (-2, 2); directrix: x = 2