Conic Section: Hyperbola

A hyperbola is the set of points in the plane for which the magnitude of the difference between the distances from two fixed points (called the focal points or foci) is a given constant, 2a. The line passing through the foci is the axis, and the points where the axis intersects the hyperbola are called the vertices. The hyperbola will have two slant asymptotes which intersect at the center of the hyperbola. The distance from the center to a focal point is \(|c|\), and \(c^2 = a^2 + b^2\). In hyperbolas, "a" is not necessarily larger than "b".
Find the vertices, foci, and asymptotes of the hyperbola whose equation is given, and then graph it.

\[ \frac{x^2}{4} - \frac{y^2}{9} = 1 \]
Find the vertices, foci, and asymptotes of the hyperbola whose equation is given, and then graph it.

\[
\frac{y^2}{16} - \frac{x^2}{25} = 1
\]
Find the vertices, foci, and asymptotes of the hyperbola whose equation is given, and then graph it.

\[16x^2 - 9y^2 = 144\]
Hyperbolas, like all other graphs, can be shifted horizontally and/or vertically. A hyperbola which is centered at \((h, k)\) would have equation

\[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1
\]

State the center, vertices, foci, and asymptotes of each hyperbola whose equation is given below. Then graph.

\[
\frac{(x-4)^2}{9} - \frac{(y-2)^2}{4} = 1 \quad \quad \quad \frac{(y+3)^2}{36} - \frac{x^2}{25} = 1
\]
Write the equation of each hyperbola in standard form and then determine the vertices, foci, and asymptotes.

\[25x^2 - 9y^2 + 50x - 36y - 236 = 0\]

\[16y^2 - 9x^2 - 32y + 36x - 164 = 0\]
If we are given information about a hyperbola, we can determine its equation.

Examples: Determine the equation of each hyperbola described below.

1) foci: $(\pm 5, 0)$, vertices: $(\pm 3, 0)$

2) vertices: $(0, \pm 3)$, foci: $(0, \pm 7)$

3) vertices: $(\pm 4, 0)$, foci: $(\pm 5, 0)$
Find the standard form of the equation of the hyperbola satisfying the given conditions.

Center: \((3, -4)\); Focus: \((10, -4)\); Vertex: \((9, -4)\)