Conic Section: Ellipses

An ellipse is the set of points in a plane for which the sum of the distances from two fixed points is a given constant, 2a. The two fixed points are called the focal points or foci of the ellipse; the line passing through the foci is the major axis. The points of intersection of the axis and the ellipse are the vertices, and the perpendicular bisector of the major axis is the minor axis.

The vertices are (±a, 0) and the foci are (±c, 0), where $c^2 = a^2 - b^2$. The "minor" vertices are (0, ±b).
If the major axis is located on the $x$-axis and the minor axis is on the $y$-axis, then the equation of the ellipse is

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \]

(Note that $a^2 > b^2$.)

The equation of the ellipse graphed below is

\[ \frac{x^2}{25} + \frac{y^2}{16} = 1 \]
If the major axis is located on the $y$-axis and the minor axis is on the $x$-axis, then the equation of the ellipse is
\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1. \] (Again, note that $a^2 > b^2$.)

The equation of the ellipse graphed below is \[ \frac{x^2}{4} + \frac{y^2}{16} = 1 \]
Given the equation of the ellipse below, determine the vertices and foci, and graph.

\[
\frac{x^2}{16} + \frac{y^2}{9} = 1
\]

\[
\frac{x^2}{25} + \frac{y^2}{36} = 1
\]
Given the equation of the ellipse below, determine the vertices and foci, and graph.

\[ 25x^2 + 16y^2 = 400 \]

\[ 36x^2 + 81y^2 = 2916 \]
Ellipses, like all other graphs, can be shifted horizontally and/or vertically. An ellipse which is centered at \((h, k)\) would have equation

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1
\]

Determine the center of each ellipse whose equation is given below, and sketch a graph, showing the direction of the major axis (horizontal or vertical). Then determine the lengths of the major and minor axes.

1. \(\frac{(x-3)^2}{1} + \frac{(y-5)^2}{4} = 1\)

2. \(\frac{(x+7)^2}{36} + \frac{y^2}{25} = 1\)

3. \(\frac{(x-1)^2}{16} + \frac{(y+4)^2}{9} = 1\)

4. \(\frac{x^2}{49} + \frac{(y-2)^2}{81} = 1\)
State the center, vertices, and foci of each ellipse whose equation is given below, and then graph.

\[
\frac{(x - 3)^2}{1} + \frac{(y - 5)^2}{4} = 1 \quad \text{and} \quad \frac{(x + 7)^2}{36} + \frac{y^2}{25} = 1
\]
State the center, vertices, and foci of the ellipse whose equation is given below, and then graph.

\[
\frac{(x + 2)^2}{49} + \frac{(y - 3)^2}{1} = 1
\]
Write the equation of each ellipse below in standard form by completing the square and then determine the center, vertices, and foci.

\[4x^2 + 9y^2 - 32x + 36y + 64 = 0\]

\[9x^2 + y^2 + 18x - 6y - 63 = 0\]
\[ x^2 + 4y^2 - 2x - 16y + 13 = 0 \]

center: (1, 2); vertices: (-1, 2), (3, 2); foci: \((1 \pm \sqrt{3}, 2)\)

\[ 4x^2 + 9y^2 - 16x + 90y + 97 = 0 \]

center: (2, -5); vertices: (-4, -5), (8, -5); foci: \((2 \pm 2\sqrt{5}, -5)\)
If we are given information about an ellipse, we can determine its equation.

Examples: Determine the equation of each ellipse described below.

1) x-intercepts: \((\pm 5, 0)\), y-intercepts: \((0, \pm 3)\)

2) vertices: \((\pm 5, 0)\), foci: \((\pm 2, 0)\)

3) vertices: \((0, \pm 6)\), foci: \((0, \pm 5)\)

4) vertices: \((-4, -2)\) and \((-4, 8)\); one focus at \((-4, 0)\)
Determine the equation of each ellipse graphed below.