3.5: Exponential Growth and Decay; Modeling Data

In this section we will be using the techniques we learned in previous sections to solve applied problems involving exponential functions.

The functions will deal with phenomena such as population growth, radioactive decay, etc., and will all be of the form \( A = A_0 e^{kt} \) where \( A_0 \) is the initial population, quantity, etc., \( k \) is the growth or decay rate, and \( t \) is time. If the quantity is increasing (or growing), \( k \) will be positive, and if the quantity is decreasing (or decaying), \( k \) will be negative.

For example, if the population of Ecalon initially was 3.4 million, and it’s growing at a continuous rate of 5.2\% annually, then the model for its population at time \( t \) is given by
\[
A = 3.4e^{0.052t}
\]

Or, if the initial quantity of radioactive Strontium is 500 grams, and it’s decaying at a continuous rate of 1.6\% every year, then the model for the quantity at time \( t \) is given by
\[
A = 500e^{-0.016t}
\]
Determine the initial quantity \( (A_0) \) and the growth or decay rate \((k)\) of each function below.

\[
A = 400e^{-0.0035t} \quad A = 5.9e^{0.067t}
\]

\[
A = 300e^{0.5t} \quad A = 60e^{-0.03t}
\]
Suppose the population of Erehwon is given by the function below.

\[ A = 7000e^{-0.023t} \]

Determine

a) the initial population (at time 0).

b) whether the population is increasing or decreasing.

c) at what rate it's increasing or decreasing.

d) the population of Erehwon after 50 years have passed.

e) the number of years after which Erehwon's population will be reduced to 100 residents.
The town of Andersonville had a population of 4500 in the year 1950, and by the year 2000, the population had risen to 22,000. Determine a formula for the population of Andersonville at time $t$ years since 1950.
The town of Smithberg had a population of 50,000 in the year 1950, but by the year 2000, the population had fallen to 20,000. Determine a formula for the population of Smithberg at time $t$ years since 1950.
The town of Nwotyna had a population of 7200 in the year 2000, and it has a continuous growth rate of 1.05%.

a) Write a formula for the population of Nwotyna at time $t$ years since 2000.

b) Predict the population of Nwotyna in the year 2025.

c) Determine the year that Nwotyna's population will be double what it was in the year 2000.
Bulgaria's population in the year 2007 was 7.3 million people, and its projected population for the year 2025 is 6.3 million.

Determine a formula for the population of Bulgaria at time $t$ years after 2007.
The half-life of a substance is the amount of time necessary for half of the substance to decay.

Suppose the half-life of aspirin is 12 hours. If you take 200 mg, how much is still in your bloodstream after 12 hours?
24 hours?
36 hours?
48 hours?

Determine a formula for the amount of aspirin in your bloodstream at time $t$.

How long will it take for the initial quantity to be reduced to 40%?
The half-life of Radium-226 is 1620 years. Determine the decay rate, \( k \), to 6 decimal places, and then write a formula for the amount of Radium-226 at time \( t \).

A certain substance decays at a continuous rate of 4.3% per hour. Determine the half-life of the substance.
Use the appropriate compound interest formula to solve each of the following:

1) $5000 is invested at an interest rate of 3%, compounded quarterly. How long must it stay in the account in order to accumulate to $8000?

2) Jay wants to have a balance of $10,000 in his account 5 years from now. If he can invest in an account that pays 2.3% compounded continuously, how much must he invest today?

3) In order for an investment of $6000 to double in 10 years, what interest rate is required, if interest is compounded
   a) annually?  
   b) continuously?