3.4: Exponential and Logarithmic Equations

Many equations involving logarithms can be converted to exponential form for ease in solving, while equations involving exponents can have a logarithm applied to them, allowing the power rule for logarithms to be applied. Some exponential equations can be solved simply by expressing each side as a power of the same base.

Ex:

\[ 7^x = \sqrt{7} \quad \quad \quad 5^{2x-3} = 25 \]

\[ 4^x = 8 \quad \quad \quad e^{3x+1} = \frac{1}{3\sqrt{e}} \]

These same equations could have been solved by applying a logarithm to both sides, and using the power rule.
If it is not possible to write both sides as a power of the same base, then they should be solved by applying a logarithm to both sides, and using the power rule.

Ex: \[ 3^x = 19 \quad \quad e^{2x-7} = 4 \]

It is important to have the exponential expression isolated before applying the logarithm to both sides.

Ex: \[ 5 \cdot 2^x - 9 = 6 \quad \quad 100e^x + 7 = 3 \]
An equation involving logarithms can be converted to exponential form for ease in solving, but before converting to exponential form, the equation should be simplified to contain only one logarithm, and the logarithm should be isolated.

Ex:

$$\log_3(2x - 7) = 2$$  \hspace{1cm}  $$\ln(3x + 5) = 0$$

$$5\ln(2x) = 20$$  \hspace{1cm}  $$\log_7(x + 3) + 9 = 10$$
Sometimes we must use one or more of the properties of logarithms to condense multiple logs into one log expression.

\[ \log_7(x + 3) + \log_7(x - 3) = 2 \]

\[ \log_5(2x + 7) - \log_5(x - 4) = 2 \]

\[ 2 \log_3(x + 4) = \log_3 9 + 2 \]

\[ 6 \log(x - 1) = 5 - \log 100 \]
The formula below models the population of Ecalpon, in millions, $t$ years after 2005. Use the formula to determine

a) the population of Ecalpon in 2005.

b) the population of Ecalpon in 2010.

c) the year that Ecalpon's population will reach 20 million.

$$P = 3.4e^{0.052t}$$
Use the appropriate compound interest formula to solve each of the following:

1) $5000 is invested at an interest rate of 3%, compounded quarterly. How long must it stay in the account in order to accumulate to $8000?

2) Jay wants to have a balance of $10,000 in his account 5 years from now. If he can invest in an account that pays 2.3% compounded continuously, how much must he invest today?

3) In order for an investment of $6000 to double in 10 years, what interest rate is required, if interest is compounded
   a) annually?  
   b) continuously?