3.2 Logarithmic Functions

For $x > 0$ and $b > 0$, $b \neq 1$,

$y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the logarithmic function with base $b$.

We can change from logarithmic to exponential form to more easily evaluate an expression.

Ex: Convert from a log to an exponential and find the value of $x$.

$$2 = \log_5 x \quad 5 = \log_2 x \quad x = \log_6 36$$

$$x = \log_2 8 \quad 2 = \log_x 49 \quad -1 = \log_\frac{1}{10} x$$
We can also change from exponential to logarithmic form.

Ex: Convert from an exponential to a logarithm.

\[ 12^2 = 144 \quad \frac{1}{16} = 4 \quad 7^{-1} = \frac{1}{49} \]

Basic Logarithmic Properties Involving One:

\[ \log_b b = 1 \] because 1 is the exponent to which \( b \) must be raised to obtain \( b \).

\[ \log_b 1 = 0 \] because 0 is the exponent to which \( b \) must be raised to obtain 1.

Examples:

\[ \log_5 5 = \quad \log_3 1 = \quad \log_{34} 34 = \]
In section 3.1, we saw that the exponential function $y = b^x$ was one-to-one, so it has an inverse. To find its inverse, we switch $x$ and $y$ and solve for $y$.

$$y = b^x$$

This result leads to the following.

**Inverse properties of Logarithms:**
For $b > 0$ and $b \neq 1$,

$$\log_b b^x = x \quad \text{and} \quad b^{\log_b x} = x$$
There are two bases that are widely used in applications, base 10, referred to as the **common logarithm**, usually denoted $\log$, and base $e$, referred to as the **natural logarithm**, denoted $\ln$.

Evaluate each of the following:

\[
\begin{align*}
\log_{10} 100 & \quad \ln e^5 & \quad \log_8 64 & \quad \log_{10} 1 \\
\ln e & \quad \log_{10} 10 & \quad \log_{3.7} 3.7 & \quad \log_{\frac{1}{5}} 1 \\
\log_7 \sqrt{7} & \quad \log_9 \frac{1}{81} & \quad \log_4 4^6 & \quad \log_3 \frac{1}{\sqrt{3}}
\end{align*}
\]
Notes on the graph of $y = \log_b x$

The domain of the function is $(0, \infty)$ and the range is $(-\infty, \infty)$.

There is a vertical asymptote at $x = 0$, and there is no horizontal asymptote.

The $x$-intercept is at 1, and there is no $y$-intercept.
Just as with the graphs of other types of functions, the graphs of logarithmic functions can have transformations applied to them, including shifts, reflections, stretches, and compressions.

Describe how the graph of each function below would differ from the graph of \( y = \log_b x \).

\[
\log_b (x - 3) \quad \log_b x - 3 \quad \log_b (-x) \\
- \log_b x \quad 2 \log_b x \quad \log_b (x + 4) + 5
\]