2.2: Quadratic Functions

A quadratic function is a function of the form \( y = ax^2 + bx + c \), where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

It can also be written in the form \( y = a(x - h)^2 + k \), where \((h, k)\) is the vertex. Recall that "a" determines whether or not the parabola is reflected across the x-axis, and whether or not it is vertically stretched or compressed from the basic \( y = x^2 \) parabola.

Every quadratic function has a vertex, an axis of symmetry (which is a vertical line through the vertex), and a y-intercept. It may have 0, 1, or 2 x-intercepts.

Note: The axis of symmetry is the vertical line \( x = h \), where \( h \) is the x-coordinate of the vertex.
Determine the vertex of each quadratic function below and sketch its graph.

1) \( y = 2(x - 3)^2 - 5 \)
2) \( y = -1(x + 2)^2 + 3 \)
3) \( y = \frac{1}{2} x^2 - 4 \)
If the equation of the quadratic function is not in vertex form, the vertex can be determined in one of two ways:

1) Put the equation into vertex form by completing the square.  
   OR

2) Use the formula at right to find the vertex.  \[
(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right))
\]

Ex: Determine the vertex of each function below.

1) \( f(x) = 2x^2 - 8x + 5 \)  
   2) \( f(x) = -x^2 + 4x - 1 \)  
   3) \( f(x) = x^2 - 7x \)
Recall: To find the y-intercept, we let $x = 0$ and solve for $y$.

To find the x-intercept(s), we let $y = 0$ and solve for $x$.

If the equation is in the form $y = ax^2 + bx + c$, the y-intercept is $(0, c)$.

To find the x-intercept(s), we must solve the equation $0 = ax^2 + bx + c$, which can be done in one of several ways:

1) Factoring

2) Square Root Property

3) Quadratic Formula

Determine the x-intercept(s) and y-intercept of each function below.

1) $y = x^2 - 5x$  
2) $y = (x + 3)^2 - 4$  
3) $y = 2x^2 + 3x - 7$
Solve each equation below by factoring.

1) $x^2 - 49 = 0$
2) $5x^2 + 10x = 0$
3) $x^2 + 12 = 7x$

Solve each equation below by using the square root property.

1) $x^2 - 64 = 0$
2) $3x^2 = 45$
3) $2(x - 5)^2 + 1 = 7$
Quadratic Formula: If \( ax^2 + bx + c = 0 \), then

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

The expression under the radical, \( b^2 - 4ac \), is called the discriminant, because it discriminates between the three possibilities of 2, 1, and 0 x-intercepts, as follows:

If \( b^2 - 4ac > 0 \), there are 2 real solutions, thus 2 x-intercepts.

If \( b^2 - 4ac = 0 \), there is 1 real (repeated) solution, thus 1 x-intercept.

If \( b^2 - 4ac < 0 \), there are no real solutions, thus 0 x-intercepts.

Solve each equation below by using the quadratic formula.

1) \( x^2 - 6x - 1 = 0 \) \hspace{1cm} 2) \( 2x^2 + 5x + 3 = 0 \) \hspace{1cm} 3) \( x^2 - x = 7 \)
Recall that the sign of "a" determines whether the parabola will be concave up or concave down. If "a" is positive, the parabola is concave up, and if "a" is negative, the parabola is concave down.

If the parabola is concave up, the vertex is a minimum (lowest point on the graph), and if the parabola is concave down, the vertex is a maximum (highest point on the graph).

Determine whether each function below has a maximum or a minimum, and state its domain and range.

1) \( f(x) = 6x^2 - 6x \)  
2) \( f(x) = -4x^2 + 8x - 3 \)
If we know the vertex and one other point on the graph of a parabola, we can write its equation.

Ex: Write the equation of a parabola with vertex (5, 2) which passes through the point (3, -1).

Ex: Write the equation of a parabola with vertex (-4, 1) which passes through the point (0, 6).
Applications of Quadratic Functions:

1) Among all pairs of numbers whose difference is 20, find a pair whose product is as small as possible. What is the minimum product?

2) You have 40 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?