1.7 Combinations of Functions; Composite Functions

We can add, subtract, multiply, and divide two functions as follows:

\[(f + g)(x) = f(x) + g(x)\]
\[(f - g)(x) = f(x) - g(x)\]
\[(fg)(x) = f(x)g(x)\]
\[(f/g)(x) = f(x)/g(x), \quad g(x) \neq 0\]

The domain of the combination of functions is the intersection of the domains of the two original functions, provided that the denominator in \(f/g\) is not 0.

Ex: Given \(f(x) = 3x + 5\) and \(g(x) = x^2 - 9\), find each of the above combinations and give their domains.
Given $f(x) = \sqrt{x + 7}$ and $g(x) = \sqrt{2x - 8}$, find each of the combined functions and state their domains.
Given \( f(x) = \frac{5}{x-3} \) and \( g(x) = \frac{3}{x+2} \), find each of the combined functions and state their domains.
The composition of two functions $f$ and $g$, denoted $f \circ g$, is found by using the function $g$ as the input in the function $f$.

Note: $(f \circ g)(x)$ means $f(g(x))$.

Ex. Given $f(x) = 3x - 7$ and $g(x) = x^2 + 5$, find $(f \circ g)(x)$.

Now find $(g \circ f)(x)$, which uses the function $f$ as the input in the function $g$. 
The domain of \((f \circ g)(x)\) is everything in the domain of \(g\) for which the composition is defined.

Ex: Given \(f(x) = 3x - 7\) and \(g(x) = \sqrt{2x + 10}\), find \((f \circ g)(x)\) and state its domain.

Now find \((g \circ f)(x)\) and state its domain.
Ex: Given \( f(x) = x^2 + 5 \) and \( g(x) = \sqrt{x+7} \), find \((f \circ g)(x)\) and \((g \circ f)(x)\) and state their domains.
We can also evaluate a composition by beginning in the inside function, getting the output, and then using it as the input in the outside function.

Ex: Given \( f(x) = 3x - 7 \) and \( g(x) = \sqrt{2x + 10} \), find

a) \( (f \circ g)(3) \)

b) \( (f \circ g)(-3) \)

c) \( (g \circ f)(3) \)

d) \( (g \circ f)(1) \)
Ex: Given \( f(x) = \frac{5}{x+4} \) and \( g(x) = \frac{1}{x} \), find \((f \circ g)(x)\) and \((g \circ f)(x)\) and state their domains.

Using \( f \) and \( g \) as above, find \((f \circ g)(3)\) and \((g \circ f)(3)\).
In addition to composing two functions into one function, we can decompose one function into two. This is referred to as decomposition, (not to be confused with what happens to rotten fruit, etc.)

Express the given function \( h \) as a composition of two functions \( f \) and \( g \) so that \( h(x) = (f \circ g)(x) \).

\[
a) \quad h(x) = (3x + 8)^3 \\
b) \quad h(x) = \sqrt[7]{x^2 - 1} \\
c) \quad h(x) = \frac{1}{2x^2 + 5}
\]