1.2: Basics of Functions and Their Graphs

A relation is any set of ordered pairs. The set of all first coordinates (inputs) is called the domain, and the set of all second coordinates (outputs) is called the range. If every element in the domain corresponds to exactly one element in the range, then the relation is a function.

A relation can be expressed as a set of ordered pairs, a table, a graph, an equation, or a verbal description.

Determine whether each relation below represents a function or not, and give the domain and range.

1) \{(7, 1), (3, -5), (0, -5)\}

2) \[
\begin{array}{c|cccc}
\text{x} & 0 & 2 & -2 & 4 \\
\text{y} & 5 & 4 & 3 & 2 \\
\end{array}
\]

3) [Graph of a curve]

4) [Graph of a curve]
To determine whether or not an equation defines $y$ as a function of $x$, solve the equation for $y$ to determine whether or not every input corresponds to exactly one output.

Determine whether or not each equation below defines $y$ as a function of $x$.

1) $x^2 + y = 7$  
2) $x + y^2 = 5$  
3) $4x + 5y = 10$  
4) $x^3 + y^2 = 49$
To evaluate a function, replace the variable in the function with the given input.

Evaluate each function below.

Given \( f(x) = 5x - 2 \), find a) \( f(3) \) b) \( f(-4) \) c) \( f(x + 1) \)

Given \( f(x) = x^2 - 3x + 7 \), find a) \( f(3) \) b) \( f(-4) \) c) \( f(x + 1) \)
You can also evaluate a function from its graph.

Use the function shown below to find

a) $f(0)$  

b) $f(2)$  

c) $f(3)$
In this section, we begin to look at transformations of functions, that is, taking basic functions and applying certain changes to them to get new functions.

For example, graph the pairs of functions below and describe how they are related.

\[ f(x) = 3x, \quad g(x) = 3x - 4 \]

\[ f(x) = x^2, \quad g(x) = x^2 + 3 \]

Use inputs -2, -1, 0, 1, 2.

\[ f(x) = \sqrt{x}, \quad g(x) = \sqrt{x + 2} \]

For \( f(x) \), use inputs 0, 1, 4, 9. For \( g(x) \), use inputs -2, -1, 2, 7.
We can write functions which model particular situations, such as the following, and then evaluate them at certain values.

1) A company which manufactures a particular product has fixed costs of $95,000 and variable costs of $50 per item produced.
   a) Write a function, \( C(x) \), which models the total cost as a function of the number of items produced.
   
   b) Find and interpret \( C(70) \).

2) A brand new building is valued at $450,000, and it depreciates by $25,000 every year.
   a) Write a function, \( V(t) \), which models the value of the building as a function of the time in years.
   
   b) Find and interpret \( V(5) \).