1.10: Modeling with Functions

Ex. 1:
A company has fixed costs of $50,000 and variable costs of $100 per unit produced.

a) Express the total cost, \( f \), as a function of the number of units produced, \( x \).

b) How many units can be produced for a total cost of $80,000?

Ex. 2:
A company pays its sales staff a salary of $20,000 per year plus a 2% commission on all sales they make.

a) Express the total annual salary, \( f \), as a function of the amount of sales, \( x \).

b) Determine the amount of sales required to earn an annual salary of $50,000.
Ex. 3:
The enrollment at a particular college was 5500 in the year 1990, and it has increased by an average of 200 students per year since then.

a) Express the enrollment, \( f \), as a function of the number of years since 1990, \( x \).

b) Determine the year in which the enrollment will reach 15,000.

Ex. 4:
In the year 1960, the percentage of high school seniors in the town of Ecalpon who had tried cigarettes was 80%. Throughout the years since then, the percentage decreased by an average of 1.25% every year.

a) Express the percentage of high school seniors, \( f \), as a function of the number of years since 1960, \( x \).

b) Determine the year in which the percentage of high school seniors trying cigarettes was or will be 24%.
Ex. 5:
You are trying to decide between two health clubs. Club A charges a one-time membership fee of $100 and a monthly charge of $25. Club B charges a one-time membership fee of $50 and a monthly charge of $31.25.

a) Express the total amount paid to Club A, \( f \), as a function of the number of months, \( x \).

b) Express the total amount paid to Club B, \( f \), as a function of the number of months, \( x \).

c) Determine the number of months of membership at which the two plans would have the same cost.
Ex. 6: 
An open box is made from a square piece of cardboard 36 inches on a side by cutting identical squares from the corners and turning up the sides.

a) Express the volume of the box, \( V \), as a function of the length of the side of the square cut from each corner, \( x \).

b) Find and interpret \( V(3) \), \( V(6) \), \( V(9) \), and \( V(12) \). What is happening to the volume of the box as the length of the side of the square cut from each corner increases?

c) Find the domain of \( V \).

d) Find the value of \( x \) which maximizes the volume.
Ex. 7:
The sum of two numbers is 45. Express the product of the two numbers, P, as a function of one of the numbers, x.

Ex. 8:
You have 400 feet of fencing to enclose a rectangular field.

a) Express the area of the field, A, as a function of one of its dimensions, x.

b) Find the value of x which maximizes the area.
Ex. 9:

John inherited $100,000 from his grandmother, and decided to invest some of it in a secure CD which pays 2% annual interest. The rest he invested in a non-insured bond which pays 12% annual interest.

a) Express the interest from both investments, $I$, as a function of the amount of money invested at 2%, $x$.

b) If John wants to earn $10,000 in interest, how much should he invest in each fund?