A rational number is the quotient of two integers. A rational expression is the quotient of two polynomial expressions.

**Definition** Rational Expression

A rational expression is an algebraic expression of the form \( \frac{P}{Q} \), where \( P \) and \( Q \) are polynomials such that \( Q \neq 0 \).

The following are examples of rational expressions:

\[
\frac{x^2 - 4x}{2x - 8}, \quad \frac{3x^2 - 2x - 8}{3x^2 - 14x - 24}, \quad \frac{x^2 y^3 + 11x^5 y^4 - 7xy}{x y^5 - x^2 y}
\]

Because division by zero is never allowed, we must only consider values of the variable for which the denominator does not equal zero. For example, the rational expression \( \frac{x^2 - 4x}{2x - 8} \) is defined for all values of \( x \) with the exception of \( x = 4 \). The value of \( x = 4 \) produces zero in the denominator and thus cannot be considered as a possible value of the variable. To simplify a rational expression, try factoring the numerator and denominator and cancel any common factors. The restrictions placed on the original expression still hold for the simplified expression.

\[
\frac{x^2 - 4x}{2x - 8} = \frac{x(x - 4)}{2(x - 4)} = \frac{x}{2}
\]
Simplify each rational expression.

\[ \frac{x^2 + |x - 12|}{x^2 + 9x + 20} \quad \frac{-12}{4, 5} \quad \frac{x^3 + 1}{x + 1} \quad \frac{x^2 - x - 2}{2x - x^2} \]

\[ \frac{(x+4)(x-3)}{(x+4)(x+5)} \quad \frac{20}{4, 5} \quad \frac{(x+1)(x^2 - x + 1)}{(x+1)} \quad \frac{-1(x-2)(x+1)}{x} \]

\[ \frac{x - 3}{x + 5} \quad x - 2 = -1(2 - x) \]
We multiply rational expressions similarly to the way we multiply rational numbers.

If \( \frac{A}{B} \) and \( \frac{C}{D} \) are rational expressions, then \( \frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD} \).

Multiply each of the following expressions, and write your answer in simplest form.

\[
\frac{18x + 18}{2x + 6} \cdot \frac{x + 3}{6x^2 - 6} = \frac{3(x+1)}{2(x+1)(x-1)} = \frac{3}{2(x-1)}
\]

\[
\frac{x^2 - 6x - 7}{2x^2 - 98} \cdot \frac{x^2 + 14x + 49}{3x^2 + 24x + 21} = \frac{1}{6}
\]

\[
\frac{(x-7)(x+1)}{2(x^2-49)} \cdot \frac{(x+7)^2}{3(x^2+8x+7)}
\]

\[
\frac{1}{2(x-7)(x+7)} \cdot \frac{1(x+7)^2}{3(x+7)(x+1)}
\]
Divide and simplify.

\[
\frac{4x}{9} \div \frac{8x+16}{9x+18}
\]

\[
\frac{x^3-8}{2x^2-x-6} \div \frac{x^2+2x+4}{6x^2+11x+3}
\]
Adding and Subtracting Rational Expressions

We add and subtract rational expressions in much the same way. We start by determining the least common denominator (LCD). The LCD is the smallest algebraic expression divisible by all denominators.

Perform the indicated operations and simplify.

a. \( \frac{3}{x+1} - \frac{2-x}{x+1} = \frac{3-(2-x)}{x+1} = \frac{3-2+x}{x+1} = \frac{1+x}{x+1} = \boxed{1} \)

b. \( \frac{3}{x^2+2x} + \frac{x-2}{x^2-x} = \frac{3(x-2)}{x(x+2)(x-1)} + \frac{(x-2)(x+2)}{x(x-1)(x+2)} = \frac{3x-3+x^2-4}{x(x+2)(x-1)} = \frac{x^2+3x-7}{x(x+2)(x-1)} \)

c. \( \frac{7}{12x^2y} - \frac{13 \cdot 3x}{4xy} = \frac{7 - 39x}{12x^2y} \)

d. \( \frac{8}{(x+8)(x-1)} - \frac{6(x-1)}{(x+8)(x-1)} = \frac{8x+64-6x+6}{(x+8)(x-1)} = \frac{2x+70}{(x+8)(x-1)} \)

LCD: \(12x^2y\)

LCD: \(x(x+2)(x-1)\)

LCD: \((x-1)(x+8)\)
Perform the indicated operations and simplify.

\[ \frac{3}{x-y} - \frac{x+5y}{x^2-y^2} \]

\[ \frac{3(x+y)}{(x-y)(x+y)} - \frac{(x+5y)}{(x+y)(x-y)} \]

\[ \frac{3x+3y - x - 5y}{(x-y)(x+y)} \]

\[ \frac{2x-2y}{(x-y)(x+y)} \]

\[ \frac{2(x-y)}{(x-y)(x+y)} \]

\[ \frac{2}{x+y} \]
Method II for Simplifying Complex Rational Expressions

Step 1. Determine the overall LCD. (For top & bottom of complex fraction)

Step 2. Multiply the numerator and denominator of the complex rational expression by the overall LCD.

Step 3. Simplify.

Simplify the complex rational expressions.

\[
\frac{7x + 1}{7x - 1}
\]

\[
\frac{\frac{4}{x-1} - \frac{5}{x-1}}{\frac{6}{x-1}}
\]

\[
\frac{4x - 4 - 5}{6 - 7x + 7}
\]

\[
\frac{4x - 9}{-7x + 13}
\]
\[
\frac{-1}{x} - \frac{3}{x+4} \times (x+4) = \frac{-1}{x} \times (x+4) - \frac{3x}{x+4} \\
\frac{2}{x+4} + \frac{2}{x} \times (x+4) = \frac{2}{x+4} + \frac{2x}{x} \\
\text{LCM: } x(x+4) \\
\frac{-x-4-3x}{2+2x+8} = \frac{-x-4}{2x+10} \\
\frac{-2}{x(x+1)} \frac{1}{x(x+5)} = \frac{-2(x+1)}{x+5}
\]