R.1 Real Numbers

A set is a collection of objects. Each object in the set is called an element or a member of the set. We typically use braces \{\} to enclose all elements of a set.

Ex: \( A = \{3, 4, 5, 6, 7, 8\} \) and \( B = \{3, 5, 7\} \)

We would say that \( B \) is a subset of \( A \) and write \( B \subset A \).

The set of real numbers consists of several subsets of numbers, as described below.

Natural numbers (also called counting numbers): \( N = \{1, 2, 3, 4, 5, \ldots\} \)

Whole numbers: \( W = \{0, 1, 2, 3, 4, 5, \ldots\} \)

Integers: \( Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \)

Rational numbers: \( Q = \left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0 \right\} \)

Examples:

\[
\frac{1}{2}, \quad \frac{-3}{5}, \quad \frac{7}{1} = 7, \quad \frac{0}{3} = 0, \quad \frac{3}{0} \text{ is undefined!}
\]

Note that \( N \subset W \subset Z \subset Q \).

Irrational numbers: \( I = \{x \mid x \text{ is not a rational number}\} \)

Examples: \( \sqrt{3}, \pi, \sqrt{2} \)

The set of real numbers, denoted \( \mathbb{R} \), is the set of all rational and irrational numbers.
Real numbers can be represented by points located on a number line called the real axis.

Classify each number in the set below as a natural number, whole number, integer, rational number, irrational number, and/or real number.

\[ \{-5, -\frac{1}{3}, 0, \sqrt{3}, 1.4, 2\pi, 11\} \]

-5: integer, rational, real
-1/3: rational, real
0: whole, integer, rational, real
\(\sqrt{3}\): irrational, real
1.4: rational, real
2\(\pi\): irrational, real
11: All except irrational
Set-builder notation describes a set of numbers, as in the following example.

\[ \{x \mid x > 7\} \]

This is read as "The set of all \( x \) such that \( x \) is greater than 7."

This same set can also be represented using a number line graph as shown below.

Additionally, it can be expressed using the interval notation \((7, \infty)\).

Note that an open circle is used to show that 7 is not included in the set. If the set included 7, we would use a closed circle and use a bracket on the interval.
Given the set sketched on the number line, a) identify the type of interval, b) write the set using set-builder notation, and c) write the set using interval notation.

\[
a) \text{half-open} \quad \left\{ x \mid -\frac{6}{5} < x \leq 3 \right\} \\
b) \left( -\frac{6}{5}, 3 \right] \\
c) \left( -\frac{6}{5}, 3 \right]
\]

In Exercises 7–10, given the set sketched on the number line, a) identify the type of interval, b) write the set using set-builder notation, and c) write the set using interval notation.

7. \[
\begin{array}{c}
0 \\
3
\end{array}
\]

8. \[
\begin{array}{c}
0 \\
\frac{1}{4}
\end{array}
\]

9. \[
\begin{array}{c}
-\frac{3}{2} \\
0 \\
2
\end{array}
\]

10. \[
\begin{array}{c}
-\sqrt{2} \\
0
\end{array}
\]

Write the set \( \left\{ x \mid -\frac{7}{2} < x \leq \pi \right\} \) in interval notation and graph the set on a number line.

\[
\left( -\frac{7}{2}, \pi \right]
\]

In Exercises 15–18, write the given set in interval notation and graph the set on a number line.

15. \( \left\{ x \mid -\frac{5}{2} \leq x \leq 1 \right\} \)

16. \( \{ x | 0 \leq x < 3 \} \)

17. \( \left\{ x \mid x = \frac{3}{4} \right\} \)

18. \( \{ x | x > -4 \} \)
We are often interested in looking at two or more sets at a time. Joining all elements of a set A with all elements of a set B is known as the union of sets A and B and is denoted as \( A \cup B \). The set of elements that are common to both sets A and B is known as the intersection of sets A and B and is written as \( A \cap B \). The set containing no elements is called the empty set and is denoted \( \emptyset \) or \( \{ \} \).

Let \( A = \{-5,0,\frac{1}{3},11,17\} \), \( B = \{-6,-5,4,17\} \) and \( C = \{-4,0,\frac{1}{4}\} \).

a. Find \( A \cup B \)    b. Find \( A \cap B \)    c. Find \( B \cap C \)

\([-5,0,\frac{1}{3},11,17, -6, 4, 17 \] \[ -5,17 \] \[ \{ \} \]

Find the Intersection of Intervals
Find the intersection of the following intervals and graph the set on a number line.

a. \([0, \infty) \cap (-\infty, 5]\)

b. \((-\infty, -2) \cup (-2, \infty) \cap [-4, \infty)\)

\([0,5]\)

\([-4,-2), (-2, \infty)\)
The absolute value of a real number $a$ is defined as the distance between $a$ and 0 on a number line and is denoted $|a|$. For example, $|-5| = 5$ because the number -5 is 5 units away from 0 on a number line.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The absolute value of a real number $a$ is defined by $</td>
<td>a</td>
</tr>
</tbody>
</table>

Because the absolute value of a number $a$ represents the distance from $a$ to 0 on a number line, it follows that $|a| \geq 0$ for any real number $a$. This property and several others are stated below.

<table>
<thead>
<tr>
<th>Properties of Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>For all real numbers $a$ and $b$,</td>
</tr>
<tr>
<td>1. $</td>
</tr>
<tr>
<td>4. $</td>
</tr>
</tbody>
</table>

Evaluate the following expressions involving absolute value.

- $|\sqrt{2}| = \sqrt{2}$
- $\left| -\frac{4}{16} \right| = \frac{4}{16}$
- $|2 - 5| = 3$
The distance between two real numbers $a$ and $b$ on a number line is defined by $|a - b|$ or $|b - a|$.

Find the distance between the numbers -5 and 3 using absolute value.

\[ | -5 - 3 | \text{ or } | 3 - (-5) | \]
\[ | -8 | \text{ or } | 8 | \]

\[ 8 \]

\[ -5 \quad 0 \quad 3 \]