7.1 Solving Systems of Linear Equations in Two Variables

**Objective 1** Verifying Solutions to a System of Linear Equations in Two Variables

**Definition** System of Linear Equations in Two Variables

A system of linear equations in two variables is the collection of two linear equations in two variables considered simultaneously. The solution to a system of equations in two variables is the set of all ordered pairs for which both equations are true.

**Example 1** Verify the Solution to a System of Linear Equations in Two Variables

Show that the ordered pair \((-1, 3)\) is a solution to the system

\[
\begin{align*}
3x - 2y &= -9 \\
x + y &= 2
\end{align*}
\]

\[
\begin{align*}
3(-1) - 2(3) &= -9 \\
-3 - 6 &= -9 \\
-1 + 3 &= 2
\end{align*}
\]

Determine if each ordered pair is a solution to the given system.

\[
\begin{align*}
3x - 2y &= -2 \\
4x + 2y &= 16
\end{align*}
\]

\[
\begin{align*}
a. (2, 4) &\quad \text{Yes!} \\
3(2) - 2(4) &= -2 \\
6 - 8 &= -2 \\
4(2) + 2(4) &= 16 \\
8 + 8 &= 16
\end{align*}
\]

\[
\begin{align*}
b. (-4, -5) &\quad \text{No!} \\
3(-4) - 2(-5) &= -2 \\
-12 + 10 &= -2 \\
4(-4) + 2(-5) &= 16 \\
-16 + -10 &\neq 16
\end{align*}
\]
The solution to a system of two linear equations involving two variables can be viewed geometrically. Because the graph of each equation of the system is a line, we can sketch the two lines and geometrically view the solution. There are only three possibilities for the graph of a linear system of equations involving two variables:

(a) Consistent, Independent
   **One solution**
   Two lines are different, having one common point. The lines have different slopes.

(b) Consistent, Dependent
   **Ininitely many solutions**
   Two lines are the same, having infinitely many common points. The lines have the same slope and same y-intercepts.

(c) Inconsistent, No solution
   Two lines are different, having no common points. The lines have the same slope but different y-intercepts.
Ex: Solve the system of equations below by substitution.

\[
\begin{align*}
y &= 5x - 4 \\
2x - y &= 9
\end{align*}
\]

\[
\begin{align*}
y &= 5\left(\frac{-5}{3}\right) - 4 \\
&= \frac{-25}{3} - 4 \\
&= \frac{-37}{3}
\end{align*}
\]

\[
\begin{align*}
x &= \frac{5}{3} \\
y &= \frac{-37}{3}
\end{align*}
\]

Example 2  Solve a System of Equations Using the Substitution Method

Solve the following system using the method of substitution:

\[
\begin{align*}
2(5-\frac{y}{3}) - 3y &= -5 \\
10 - 2y - 3y &= -5 \\
-10 - 5y &= -15 \\
y &= \frac{5}{5} \\
y &= 1
\end{align*}
\]

\[
\begin{align*}
x &= 5 - \frac{y}{3} \\
x &= 5 - 3 \\
x &= 2
\end{align*}
\]

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Ex: Solve each system of equations below by elimination.

\[
\begin{align*}
3x + 4y &= 24 \\
2x - 4y &= -4
\end{align*}
\]

\[
\frac{5x}{5} = \frac{20}{5} \\
x = 4
\]

\[3(4) + 4y = 24\]
\[-12 + 4y = 24\]
\[4y = 12\]
\[y = 3\]

\[
\begin{align*}
2x + 3y &= 15 \\
3x - y &= 17
\end{align*}
\]

\[
\begin{align*}
2x + 3y &= 15 \\
9x - 3y &= 51
\end{align*}
\]

\[
\begin{align*}
x &= \frac{66}{11} \\
x &= 6
\end{align*}
\]

\[
\begin{align*}
12 + 3y &= 15 \\
3y &= 3 \\
y &= 1
\end{align*}
\]
Example 3 Solve a System of Equations Using the Elimination Method

Solve the following system using the method of elimination:

\[
\begin{align*}
3 \left[ -2x + 5y = 29 \right] & \rightarrow -6x + 15y = 87 \\
2 \left[ 3x + 2y = 4 \right] & \rightarrow 6x + 4y = 8
\end{align*}
\]

\[
\begin{align*}
3x + 2(5) &= 4 \\
3x + 10 &= 4 \\
3x &= -6 \\
x &= -2
\end{align*}
\]

\[
\begin{align*}
19y &= 95 \\
y &= 5
\end{align*}
\]

Solve the system by substitution or elimination.

\[
\begin{align*}
\begin{bmatrix}
16 & 4 \\
3 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
&= \begin{bmatrix}
3 \\
1
\end{bmatrix} \\
\begin{bmatrix}
-32 & 12 \\
-3 & 8
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
&= \begin{bmatrix}
9 \\
6
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
0 + 8y &= 6 \\
ym &= \frac{3}{4}
\end{align*}
\]

\[
\begin{align*}
x &= 0
\end{align*}
\]
Using Technology

We can solve the system in Example 3 using a graphing utility by first solving both equations for $y$:

$-2x + 5y = 29 \quad \rightarrow \quad y_1 = \frac{2}{5}x + \frac{29}{5}$

$3x + 2y = 4 \quad \rightarrow \quad y_2 = -\frac{3}{2}x + 2$

Graph $y_1$ and $y_2$, and then use the INTERSECT feature to find that the point of intersection is $(-2.5, 5)$.

$y = \frac{2}{3}x - 7$

$y = -\frac{4}{5}x + 2$
**Example 4**  Solve a System with No Solution

Solve the system

\[
\begin{align*}
2[x - 2y &= 11] &\rightarrow 2x - 4y = 22 \\
-2x + 4y &= 8
\end{align*}
\]

No solution

\[0 \neq 30\]

**Example 5**  Solve a System Having Infinitely Many Solutions

Solve the system

\[
\begin{align*}
-3x + 6y &= 9 \\
x - 2y &= -3
\end{align*}
\]

\[-3x + 6y = 9 \rightarrow 3x - 6y = -9\]

\[0 = 0\]

Infinitely many solutions

\[
\{(x, y) \mid x = 2y - 3\}
\]

or \((2y - 3, y)\)
Use the substitution method or the elimination method to solve the system.

\[
\begin{align*}
\frac{35}{7} - \frac{1}{4} &= \frac{1}{3} \cdot \frac{35}{7} \\
\frac{5}{7} - 4y &= 1 \cdot \frac{7}{1} \\
\end{align*}
\]

\[5x + 28y = -7\]
\[5x - 28y = 7\]

\[0 = 0\]

Infinitely many solutions

\[-28y = -5x + 7\]
\[-280 = -28 -28\]
\[y = \frac{5}{28}x - \frac{1}{4}\]
OBJECTIVE 4 Solving Applied Problems Using a System of Linear Equations

Five-Step Strategy for Problem Solving Using Systems of Equations

Step 1. Read the problem several times until you have an understanding of what is being asked. If possible, create diagrams, charts, or tables to assist you.

Step 2. Choose variables that describe each unknown quantity. You got 2 variables!

Step 3. Write a system of equations using the given information and the variables.

Step 4. Carefully solve the system of equations using the method of elimination or substitution.

Step 5. Make sure that you have answered the question, and check all answers to ensure they make sense.

Example 6 Determine the Number of Touchdowns Thrown

Roger Staubach and Terry Bradshaw were both quarterbacks in the National Football League. In 1973, Staubach threw three touchdown passes more than twice the number of touchdown passes thrown by Bradshaw. If the total number of touchdown passes between Staubach and Bradshaw was 33, how many touchdown passes did each player throw?

\[
\begin{align*}
X &= \text{Bradshaw's passes} = 10 \\
Y &= \text{Staubach's passes} = 23 \\
X + Y &= 33 \\
Y &= \frac{3 + 2X}{3} = 3 + 2(10) = 23 \\
X + 3 + 2X &= 33 \\
3X + 3 &= 33 \\
3X &= 30 \\
X &= 10
\end{align*}
\]
Example 7  Find the Number of Tickets Sold at a Jazz Festival
During one night at the jazz festival, 2,100 tickets were sold. Adult tickets sold for $12, and child
tickets sold for $7. If the receipts totaled $22,100, how many of each type of ticket were sold?

\[
\begin{align*}
x &= \text{adult tickets} = 1480 \\
y &= \text{child tickets} = 620 \\
-7(x + y) &= 2100 \\
-7x - 7y &= -14,700 \\
12x + 7y &= 22,100 \\
\frac{5x}{5} &= \frac{7400}{5} \\
x &= 1480
\end{align*}
\]

Example 8  Mix Beans in a Food Plant
Twin City Foods, Inc., created a 10-lb bean mixture that sells for $5.75 by mixing lima beans
and green beans. If lima beans sell for $0.70 per pound and green beans sell for $0.50 per
pound, how many pounds of each bean went into the mixture?

\[
\begin{align*}
x &= \text{lima beans, lbs. of} = 3.75 \\
y &= \text{green beans, lbs. of} = 6.25 \\
x + y &= 10 \\
0.70x + 0.50y &= 5.75 \\
0.70(10 - y) + 0.50y &= 5.75 \\
-0.70y + 0.50y &= 5.75 - 7.00 \\
-0.20y &= -1.25 \\
y &= \frac{-1.25}{-0.20} \\
y &= 6.25
\end{align*}
\]
Scott invested a total of $9500 at two separate banks. One bank pays simple interest of 12% per year while the other pays simple interest at a rate of 8% per year. If Scott earned $996.00 in interest during a single year, how much did he have on deposit in each bank?

\[
\begin{align*}
X &= \text{amt. at } 12\% = 5900 \\
Y &= \text{amt. at } 8\% = 3600 \\
X + Y &= 9500 \\
0.12X + 0.08Y &= 996 \\
0.12X + 0.08(9500 - X) &= 996 \\
0.12X + 760 - 0.08X &= 996 \\
0.04X + 760 &= 996 \\
-760 &= -760 \\
0.04X &= 236 \\
\frac{0.04X}{0.04} &= \frac{236}{0.04} \\
X &= 5900
\end{align*}
\]
Jim has $5.95 in dimes and nickels. He has a total of 82 coins. How many of each coin does he have?

\[ \begin{align*}
X &= \text{# of dimes} = 37 \\
y &= \text{# of nickels} = 45
\end{align*} \]

\[
\begin{align*}
-0.05 \left[ X + y = 82 \right] &\quad \rightarrow -0.05X - 0.05y = -4.10 \\
0.10X + 0.05y &= 5.95 \\
0.05X &= 1.85 \\
X &= \frac{1.85}{0.05} \\
X &= 37
\end{align*}
\]