Objective 1: Using Vertical Shifts to Graph Functions

Sketch the graphs of \( f(x) = |x| \) and \( g(x) = |x| + 2 \).

Graph the following functions.

\[ y = x^2 \]
\[ y = x^2 + 3 \]
\[ y = x^2 - 3 \]
**Vertical Shifts of Functions**

If $c$ is a positive real number,

The graph of $y = f(x) + c$ is obtained by shifting the graph of $y = f(x)$ vertically upward $c$ units.

The graph of $y = f(x) - c$ is obtained by shifting the graph of $y = f(x)$ vertically downward $c$ units.

Discuss the graphs of the following functions.

\[
\begin{align*}
y &= \sqrt{x} & y &= \sqrt{x} + 2 & y &= \sqrt{x} - 5
\end{align*}
\]
OBJECTIVE 2  Using Horizontal Shifts to Graph Functions

Sketch the graphs of \( f(x) = x^2 \) and \( g(x) = (x + 2)^2 \).

Graph the following functions.

\[
\begin{align*}
y &= x^2 \\
y &= (x + 3)^2 \\
y &= (x - 3)^2
\end{align*}
\]
**Horizontal Shifts of Functions**

If $c$ is a positive real number,

The graph of $y = f(x + c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the left $c$ units.

The graph of $y = f(x - c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the right $c$ units.

Discuss the graphs of the following functions.

- $y = |x|$
- $y = |x + 5|$
- $y = |x - 6|$
Use the graph of \( y = x^3 \) to sketch the graph of \( g(x) = (x - 1)^3 + 2 \).

Shifts right 1 unit up 2
OBJECTIVE 3 Using Reflections to Graph Functions

Given the graph of \( y = f(x) \), what does the graph of \( y = -f(x) \) look like?

Using a graphing utility with \( y_1 = x^2 \) and \( y_2 = -x^2 \), we can see that the graph of \( y_2 = -x^2 \) is the graph of \( y_1 = x^2 \) reflected about the \( x \)-axis.

Reflections of Functions about the \( x \)-Axis

The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) about the \( x \)-axis.
Discuss the graphs of the following functions.

\[ f(x) = -x^3 \quad f(x) = -\sqrt{x} \quad g(x) = -|x| \]
Functions can also be reflected about the $y$-axis. Given the graph of $y = f(x)$, the graph of $y = f(-x)$ will be the graph of $y = f(x)$ reflected about the $y$-axis. Using a graphing utility, we illustrate a $y$-axis reflection by letting $y_1 = \sqrt{x}$ and $y_2 = \sqrt{-x}$. You can see that the functions are mirror images of each other about the $y$-axis.

**Reflections of Functions about the $y$-Axis**

The graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ about the $y$-axis.
Use the graph of the basic function $y = \sqrt[3]{x}$ to sketch each graph.

a. $g(x) = -\sqrt[3]{x} - 2$

b. $h(x) = \sqrt[3]{1 - x}$. 
OBJECTIVE 4  Using Vertical Stretches and Compressions to Graph Functions

Use the graph of \( f(x) = x^2 \) to sketch the graph of \( g(x) = 2x^2 \).

Use the graph of \( f(x) = x^2 \) to sketch the graph of \( g(x) = \frac{1}{2}x^2 \).
**Vertical Stretches and Compressions of Functions**

Suppose $a$ is a positive real number:

The graph of $y = af(x)$ is obtained by multiplying each $y$-coordinate of $y = f(x)$ by $a$. If $a > 1$, the graph of $y = af(x)$ is a vertical stretch of the graph of $y = f(x)$. If $0 < a < 1$, the graph of $y = af(x)$ is a vertical compression of the graph of $y = f(x)$.

**Discuss the graphs of the following functions.**

\[
\begin{align*}
y &= \sqrt{x} \\
y &= 3\sqrt{x} \\
y &= \frac{1}{2}\sqrt{x}
\end{align*}
\]
Objective 5
Using Horizontal Stretches and Compressions to Graph Functions

The final transformation to discuss is a horizontal stretch or compression. A function, \( y = f(x) \), will be horizontally stretched or compressed when \( x \) is multiplied by a positive number, \( a \neq 1 \), to obtain the new function, \( y = f(\alpha x) \).

**Horizontal Stretches and Compressions of Functions**

If \( a \) is a positive real number,

- For \( a > 1 \), the graph of \( y = f(ax) \) is obtained by dividing each \( x \)-coordinate of \( y = f(x) \) by \( a \). The resultant graph is a horizontal compression.

- For \( 0 < a < 1 \), the graph of \( y = f(ax) \) is obtained by dividing each \( x \)-coordinate of \( y = f(x) \) by \( a \). The resultant graph is a horizontal stretch.
Use the graph of $f(x) = \sqrt{x}$ to sketch the graphs of $g(x) = \sqrt{4x}$ and $h(x) = \frac{1}{\sqrt{4}x}$. 
OBJECTIVE 6 Using Combinations of Transformations to Graph Functions

Use transformations to sketch the graph of \( f(x) = -2(x + 3)^2 - 1 \).
Use the graph of \( y = f(x) \) to sketch each of the following functions.

(a) \( y = -f(2x) \)  
(b) \( y = 2f(x - 3) - 1 \)

(c) \( y = \frac{1}{2}f(2 - x) + 3 \)
Summary of Transformation Techniques

Given a function \( y = f(x) \) and a constant \( c > 0 \):

1. The graph of \( y = f(x) + c \) is obtained by shifting the graph of \( y = f(x) \) vertically upward \( c \) units.

2. The graph of \( y = f(x) - c \) is obtained by shifting the graph of \( y = f(x) \) vertically downward \( c \) units.

3. The graph of \( y = f(x + c) \) is obtained by shifting the graph of \( y = f(x) \) horizontally to the left \( c \) units.

4. The graph of \( y = f(x - c) \) is obtained by shifting the graph of \( y = f(x) \) horizontally to the right \( c \) units.

5. The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) about the \( x \)-axis.

6. The graph of \( y = f(-x) \) is obtained by reflecting the graph of \( y = f(x) \) about the \( y \)-axis.

7. Suppose \( a \) is a positive real number. The graph of \( y = af(x) \) is obtained by multiplying each \( y \)-coordinate of \( y = f(x) \) by \( a \).
   - If \( a > 1 \), the graph of \( y = af(x) \) is a vertical stretch of the graph of \( y = f(x) \).
   - If \( 0 < a < 1 \), the graph of \( y = af(x) \) is a vertical compression of the graph of \( y = f(x) \).

8. Suppose \( a \) is a positive real number. The graph of \( y = f(ax) \) is obtained by dividing each \( x \)-coordinate of \( y = f(x) \) by \( a \).
   - If \( a > 1 \), the graph of \( y = f(ax) \) is a horizontal compression of the graph of \( y = f(x) \).
   - If \( 0 < a < 1 \), the graph of \( y = f(ax) \) is a horizontal stretch of the graph of \( y = f(x) \).