2.2 Circles

A circle is the set of all points \((x, y)\) in the Cartesian plane that are a fixed distance, \(r\), from a fixed point, \((h, k)\). The fixed distance, \(r\), is called the radius of the circle, and the fixed point \((h, k)\), is called the center of the circle. See Figure 11. To derive the equation of a circle, we use the distance formula that was discussed in Section 2.1.

\[
d = \sqrt{(x-h)^2 + (y-k)^2}
\]

\[
(r)^2 = (\sqrt{(x-h)^2 + (y-k)^2})^2
\]

\[
(x-h)^2 + (y-k)^2 = r^2
\]

Standard form of the equation of a circle w/ center \((h, k)\) & radius \(r\).
The **standard form of an equation of a circle** with center, \((h, k)\), and radius, \(r\), is

\[(x - h)^2 + (y - k)^2 = r^2.\]

The standard form of an equation of a circle centered at the origin with radius, \(r\), is

\[x^2 + y^2 = r^2.\]

**Objective 1** Writing the Standard Form of an Equation of a Circle

Find the standard form of the equation of the circle whose center is \((-2, 3)\) and with radius 6.

\[
(x - (-2))^2 + (y - 3)^2 = 6^2 \rightarrow (x + 2)^2 + (y - 3)^2 = 36
\]

Write the standard form of the equation of each circle described.

Center \((-2, 3), r = 4\) \hspace{1cm} Center \((1, -4), r = 3\)

\[
(x + 2)^2 + (y - 3)^2 = 16 \hspace{1cm} (x - 1)^2 + (y + 4)^2 = 9
\]
Find the standard form of the equation of the circle whose center is (0, 6) and that passes through the point (4, 2).

\[ r = \sqrt{(4-0)^2 + (2-6)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16 + 16} = \sqrt{32} \]
\[ (x-0)^2 + (y-6)^2 = (\sqrt{32})^2 \]
\[ x^2 + (y-6)^2 = 32 \]

Find the standard form of the equation of the circle that contains endpoints of a diameter at (−4, −3) and (2, −1).

Center = midpoint = \( \left(\frac{-4+2}{2}, \frac{-3+(-1)}{2}\right) = \left(-\frac{2}{2}, -\frac{4}{2}\right) = (-1, -2) \)

\[ r = \sqrt{(-1+4)^2 + (-2+1)^2} = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10} = r \]
\[ (x+1)^2 + (y+2)^2 = 10 \]

Write the standard form of the equation of each circle described.

Center (−4, 1) and tangent to the y-axis

\[ (x+4)^2 + (y-1)^2 = 16 \]
**OBJECTIVE 2** Sketching the Graph of a Circle

Find the center and the radius, and sketch the graph of the circle \((x - 1)^2 + (y + 2)^2 = 9\). Also find any intercepts.

**Center:** \((1, -2)\)

**Radius:** \(r = 3\)

To find x-intercepts, let \(y = 0\) and solve for \(x\).

\[
(x - 1)^2 + (0 + 2)^2 = 9
\]

\[
(x - 1)^2 + 4 = 9
\]

\[
(x - 1)^2 = 5 \rightarrow x - 1 = \pm \sqrt{5} + 1
\]

\[
x = 1 \pm \sqrt{5}
\]

To find y-intercepts, let \(x = 0\) and solve for \(y\).

\[
(0 - 1)^2 + (y + 2)^2 = 9
\]

\[
(-1)^2 + (y + 2)^2 = 9 \rightarrow y + (y + 2)^2 = 9
\]

\[
(y + 2)^2 = 8 \rightarrow y + 2 = \pm \sqrt{8}
\]

\[
y = -2 \pm \sqrt{8} = -2 \pm 2\sqrt{2}
\]
Find the center, radius, and intercepts of each circle and then sketch the graph.

\[ x^2 + (y - 2)^2 = 4 \]

Center: \((0, 2)\); \(r = 2\)

\[ (x - 4)^2 + (y + 7)^2 = 12 \]

Center: \((4, -7)\); \(r = \sqrt{12} = 2\sqrt{3} \approx 3.46\)

No intercepts!