The rectangular (or Cartesian) coordinate system is made up of the horizontal \((x)\) axis and the vertical \((y)\) axis, which intersect at the origin and create four quadrants, as shown in the figure below. Every point in the rectangular coordinate system can be described by an ordered pair \((x, y)\).
Now sketch the graph of $y = 3x - 7$ by plotting points.
Now sketch the graph of $4y = x^3$ by plotting points.

$$y = \frac{x^3}{4}$$

$$y = \frac{(-2)^3}{4} = \frac{-8}{4}$$

$$y = \frac{(-1)^3}{4} = \frac{-1}{4}$$

$$y = \frac{0^3}{4} = 0$$

$$y = \frac{2^3}{4} = \frac{3}{4}$$

$$y = \frac{3^3}{4}$$

(-2, -2)
(-1, -1/4)
(0, 0)
(1, 1/4)
(2, 2)
3, 6.75
Example 3  Determine Whether a Point Lies on the Graph of an Equation

Determine whether the following ordered pairs lie on the graph of the equation $x^2 + y^2 = 1$.

a. $(0, -1)$ Yes  

b. $(1, 0)$ Yes  

c. $\left(\frac{1}{3}, \frac{2}{3}\right)$ No  

d. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ Yes

Substitute the given coordinates for $x$ and $y$ in the equation. If it makes a true statement, then the point is on the graph.

a) $0^2 + (-1)^2 = 1$  

b) $1^2 + 0^2 = 1$  

$0 + 1 = 1 \checkmark$  

$1 + 0 = 1 \checkmark$

c) $\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \neq 1$  

d) $\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

$\frac{1}{9} + \frac{4}{9} \neq 1$  

$\frac{5}{9} \neq 1$

$\frac{2}{4} + \frac{2}{4} = 1$

$\frac{4}{4} = 1 \checkmark$

$(-\sqrt{2})^2 = (-\sqrt{2})(+\sqrt{2})$
**Objective 3** Finding the Midpoint of a Line Segment Using the Midpoint Formula

Suppose we want to find the midpoint $M(x, y)$ of the line segment between the points $A(x_1, y_1)$ and $B(x_2, y_2)$. See Figure 5. To find this midpoint, we simply average the $x$- and $y$-coordinates, respectively. In other words, the $x$-coordinate of the midpoint is $\frac{x_1 + x_2}{2}$, whereas the $y$-coordinate of the midpoint is $\frac{y_1 + y_2}{2}$.

$$M \text{pt: } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Example 4** Find the Midpoint of a Line Segment

Find the midpoint of the segment whose endpoints are $(-3, 2)$ and $(4, 6)$.

$$\text{Midpoint} : \left( \frac{-3 + 4}{2}, \frac{2 + 6}{2} \right) = \left( \frac{1}{2}, 4 \right)$$
Example 5  Application of the Midpoint Formula

In geometry, it can be shown that four points in a plane form a parallelogram if the two diagonals of the quadrilateral formed by the four points bisect each other. Do the points A(0, 4), B(3, 0), C(9, 1), and D(6, 5) form a parallelogram?

\[
\text{mdpt: } \left( \frac{3+6}{2}, \frac{0+5}{2} \right) = \left( \frac{9}{2}, \frac{5}{2} \right)
\]

\[
\text{mdpt: } \left( \frac{0+9}{2}, \frac{4+1}{2} \right) = \left( \frac{9}{2}, \frac{5}{2} \right)
\]

Yes!
Ex. Plot the points (0, 0) and (4, 5), and use the Pythagorean theorem to find the distance between them.

\[ \text{dist.} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]

\[ 4^2 + 5^2 = c^2 \]

\[ 16 + 25 = c^2 \]

\[ 41 = c^2 \rightarrow c = \sqrt{41} \]
Distance Formula
The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula
\[ d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \]

Ex. Use the distance formula to find the distance between the points (0, 0) and (4, 5).

\[
dist = \sqrt{(4-0)^2 + (5-0)^2} = \sqrt{16 + 25} = \sqrt{41}
\]

Example 6  Use the Distance Formula to Find the Distance between Two Points
Find the distance, $d(A, B)$, between the points $A$ and $B$.
$A(-1, 5); B(4, -5)$

\[
dist = \sqrt{(4-(-1))^2 + (-5-5)^2} = \sqrt{5^2 + (-10)^2} = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}
\]
Example 7  Application of the Distance Formula

Verify that the points $A(3, -5)$, $B(0, 6)$, and $C(5, 5)$ form a right triangle.

\[
a = \sqrt{(5-0)^2 + (5-6)^2} = \sqrt{5^2 + (-1)^2} = \sqrt{26}
\]

\[
b = \sqrt{(3-5)^2 + (-5-5)^2} = \sqrt{(-2)^2 + (-10)^2} = \sqrt{104}
\]

\[
c = \sqrt{(0-3)^2 + (6-(-5))^2} = \sqrt{(-3)^2 + 11^2} = \sqrt{9+121} = \sqrt{130}
\]

\[
a^2 + b^2 = c^2
\]

\[
(\sqrt{26})^2 + (\sqrt{104})^2 = (\sqrt{130})^2
\]

\[
26 + 104 = 130
\]

\[
130 = 130 \checkmark
\]
2.1 MML + Quiz #5!