To solve an absolute value equation, first isolate the absolute value expression, and then determine the next step, based on the following:

If $|x| = k$, where $k$ is positive, then $x = k$ or $x = -k$.

If $|x| = k$, where $k$ is zero, then $x = 0$.

If $|x| = k$, where $k$ is negative, then there is no solution.

Examples: Solve each absolute value equation.

a) $|2x - 1| = 9$

b) $|3x + 7| = 0$

c) $|4x + 9| = -5$

No Solution
Examples: Solve each absolute value equation.

a) \(|1 - 3x| = 4\)
\[
-3x = 4 \quad \text{or} \quad -3x = -4
\]
\[
x = - \frac{4}{3} \quad \text{or} \quad x = \frac{4}{3}
\]

b) \(|6x + 7| - 3 = 3\)
\[
6x + 7 = 6 \quad \text{or} \quad 6x + 7 = -6
\]
\[
x = \frac{1}{3} \quad \text{or} \quad x = -\frac{13}{6}
\]

c) \(\frac{3x + 1}{5} = \frac{2}{3}\)
\[
3(3x + 1) = 10 \quad \text{or} \quad 3(3x + 1) = -10
\]
\[
x = \frac{7}{9} \quad \text{or} \quad x = -\frac{13}{9}
\]

d) \(\frac{5x - 7}{2} + 7 = 2\)
\[
\frac{5x - 7}{2} = -5
\]
\[
x = -3 \quad \text{or} \quad x = 5
\]

No solution

f) \(|x^2 - 7| = 2\)
\[
x^2 - 7 = 2 \quad \text{or} \quad x^2 - 7 = -2
\]
\[
x = \pm \sqrt{9} \quad \text{or} \quad x = \pm \sqrt{5}
\]

No solution
Objective 2: Solving an Absolute Value “Less Than” Inequality

\[ |x| < 5 \]

All #s whose distance from 0 is less than 5

These values are all less than five units from zero.

If \(|x| < 5\), then \(-5 < x < 5\).
The solution set is \(|x| = -5 < x < 5\) in set-builder notation or \((-5, 5)\) in interval notation.

\[ |u| < c \text{ must be positive!} \]

\[ |u| < c \text{ is equivalent to } -c < u < c. \]

Solve \[ |4x - 3| + 2 \leq 7. \]

\[ |4x - 3| \leq 5 \]

\[ -5 \leq 4x - 3 \leq 5 \]

\[ -5 + 3 \leq 4x \leq 5 + 3 \]

\[ -2 \leq 4x \leq 8 \]

\[ -\frac{1}{2} \leq x \leq 2 \]

Isolate the abs. value expression.
Solve each inequality algebraically.

\[|3 - 4x| < 11\]

\[-11 < 3 - 4x < 11\]

\[-3 - 8 < -4x < -3\]

\[-14 < -4x < 8\]

\[-\frac{7}{4} > x > -2\]

\[\frac{7}{2} > x > -2\]

\[(-2, \frac{7}{2})\]

\[|3x - 7| \leq 0\]

\[3x - 7 = 0\]

\[\frac{3x}{3} = \frac{7}{3}\]

\[x = \frac{7}{3}\]
OBJECTIVE 3 Solving an Absolute Value "Greater Than" Inequality

\[ |x| > 5 \]
Numbers whose distance from 0 is more than 5.

These values are more than five units from zero.

These values are more than five units from zero.

\[ \begin{array}{c}
-5 & 0 & 5 \\
\end{array} \]

If \(|x| > 5\), then \(x < -5\) or \(x > 5\).

The solution set is \(\{x | x < -5 \text{ or } x > 5\}\) in set-builder notation or \((-\infty, -5) \cup (5, \infty)\) in interval notation.

\[ |u| > c \] is equivalent to \(u < -c\) or \(u > c\).

Solve \(|5x + 1| > 3\).

\[
\begin{align*}
5x + 1 > 3 & \quad \text{or} \quad 5x + 1 < -3 \\
-1 & \quad -1 \\
\frac{5x}{5} & > \frac{2}{5} \quad \frac{5x}{5} < \frac{-4}{5} \\
x > \frac{2}{5} \quad \text{or} \quad x < -\frac{4}{5}
\end{align*}
\]

\((-\infty, -\frac{4}{5}) \cup (\frac{2}{5}, \infty)\)
Solve each inequality algebraically.

\[ |7x - 1| \geq 8 \]
\[ 7x - 1 \leq -8 \quad \text{OR} \quad 7x - 1 \leq 8 \]
\[ \frac{8x}{7} \leq \frac{9}{7} \quad \text{OR} \quad \frac{8x}{7} \leq \frac{-7}{7} \]
\[ x \leq \frac{9}{7} \quad \text{OR} \quad x \leq -1 \]
\[ \left[ \frac{9}{7}, \infty \right) \cup (-\infty, -1] \]

\[ |3 - 5x| > 2 \]
\[ \frac{-3}{5}x > -\frac{1}{5} \quad \text{OR} \quad \frac{-3}{5}x < -\frac{2}{5} \]
\[ x < \frac{1}{5} \quad \text{OR} \quad x > 1 \]

\[ |5x - 1| + 7 \leq 9 \]
\[ 5x - 1 \leq 2 \]
\[ -2 \leq 5x - 1 \leq 2 \]
\[ x \leq \frac{3}{5} \]
\[ x \geq \frac{1}{5} \]
\[ -\frac{1}{5} \leq x \leq \frac{3}{5} \]

\[ |7x - 5| > 0 \quad \text{All real #s except } 5/7 \]
\[ 7x - 5 > 0 \quad \text{OR} \quad 7x - 5 < 0 \]
\[ x > \frac{5}{7} \quad \text{OR} \quad x < \frac{5}{7} \]