1.5 Applications of Quadratic Equations

In Section 1.2, we learned how to solve applied problems involving linear equations. In this section, we learn to solve applications that involve quadratic equations. We follow the same four-step strategy for solving applied problems that is discussed in Section 1.2. The four steps are outlined as follows.

**Four-Step Strategy for Problem Solving**

1. **Step 1.** Read the problem (several times) until you have an understanding of what is being asked. If possible, create diagrams, charts, or tables to assist you in your understanding.
2. **Step 2.** Pick a variable that describes the unknown quantity that is to be found. All other quantities must be written in terms of that variable. Write an equation using the given information and the variable.
3. **Step 3.** Carefully solve the equation.
4. **Step 4.** Make sure that you have answered the question and then check all answers to make sure they make sense.

\[ ax^2 + bx + c = 0 \]
Example 1: Find Two Numbers

The product of a number and 1 more than twice the number is 16. Find the two numbers.

\[
\begin{align*}
x &= \text{the 1st #} = -\frac{3}{2} \text{ or } 4 \\
2x + 1 &= \text{the 2nd #} = 2(-\frac{3}{2}) + 1 = -8 \text{ or } 9 \\
ac &= -\frac{72}{9}, -8 \\
x(x+1) &= 36 \Rightarrow 2x^2 + x = 36 \Rightarrow 2x^2 + x - 36 = 0 \\
2x^2 + 9x - 8x - 36 &= 0 \\
x(2x+9) - 4(2x+9) &= 0 \Rightarrow (2x+9)(x-4) = 0 \\
2x+9 &= 0 \Rightarrow x = -\frac{9}{2} \\
x-4 &= 0 \Rightarrow x = 4
\end{align*}
\]

Example: The square of a positive number added to twice the number is 8.
Find the number.

\[
\begin{align*}
x &= \text{the #} = 2 \\
x^2 + 2x &= 8 \Rightarrow x^2 + 2x - 8 = 0 \\
(x+4)(x-2) &= 0 \\
x+4 &= 0 \Rightarrow x = -4 \\
-4-4 &= 0 \\
x = 2 &\quad \text{ is the solution.}
\end{align*}
\]

Example: Three consecutive odd integers are such that the square of the third integer is 15 more than the sum of the squares of the first two. Find the integers.

\[
\begin{align*}
x &= \text{1st integer} \\
x+2 &= \text{2nd integer} \\
x+4 &= \text{3rd integer} \\
(x+4)^2 &= x^2 + (x+2)^2 + 15 \\
x^2 + 8x + 16 &= x^2 + x^2 + 4x + 4 + 15 \\
x^2 + 8x + 16 &= 2x^2 + 4x + 19 \\
-x^2 - 8x - 16 &= -x^2 - 8x - 16 \\
0 &= x^2 - 4x + 3 \\
(x-3)(x-1) &= 0 \\
x &= 3 \quad \text{or } x = 1
\end{align*}
\]
Example: Benjamin threw a rock straight up from a cliff that was 24 feet above the water. If the height of the rock, $h$, in feet, after $t$ seconds is given by the equation $h = -16t^2 + 20t + 24$, how long will it take for the rock to hit the water?
Example: The length of a rectangle is 1 inch less than twice the width. If the diagonal is 2 inches more than the length, find the dimensions of the rectangle.

\[ x = \text{width} = 8'' \\
2x - 1 = \text{the length} = 15'' \\
2x + 1 = \text{the diagonal} = 17'' \]

\[ x^2 + (2x-1)^2 = (2x+1)^2 \]
\[ x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1 \]
\[ 5x^2 - 4x + 1 = 4x^2 + 4x + 1 \]
\[ -4x^2 - 4x - 1 = -4x^2 - 4x - 1 \]
\[ x^2 - 8x = 0 \]
\[ x(x-8) = 0 \]
\[ x = 0 \]
\[ x - 8 = 0 \]
\[ x = 8 \]
Example: Imogene's car traveled 280 miles, averaging a certain speed. If the car had gone 5 mph faster, the trip would have taken 1 hour less. Find the average speed.