Calculus I Exam 2 Review (Ch. 2.1-2.7)

Review problems from pp. 140-143: Be familiar with the concepts on p. 140: 1-8, 9a-d; work the following exercises from pp. 141-143: 1, 4-6, 10-17, 19, 20, 23, 26, 41-43, 45-51, 63, and 65, plus the following problems. In addition to the problems listed from the chapter review and these 30 problems, all of your homework and quiz problems are fair game for the test as well. Please be sure to have your homework (2.1-2.6) completed and ready to hand in on Thursday.

1. Carefully and accurately state the precise mathematical definition of the derivative $f'(x)$ of a function $y = f(x)$.

2. Use the definition of the derivative to calculate $f'(x)$ for the following function.

   \[ f(x) = 5x^2 + 3x \]

3. Write the equation of the tangent line to the graph of $f(x) = 5x^2 + 3x$ at the point (-1, 2).

4. Write the equation of the normal line to the graph of $f(x) = 5x^2 + 3x$ at the point (-1, 2).

5. Find the derivative of $y = \cot x$ by first re-writing it as the appropriate trig ratio and then using the quotient rule.

Find $f'(x)$ for each of the following. Use clear notation, and do not leave any negative exponents in your answers.

6. $f(x) = 5x^5 + 4x^3 + 3x$  
7. $f(x) = \sqrt[3]{x} - \frac{6}{x}$
8. \( f(x) = 10x + 7^5 \)

9. \( f(x) = \sqrt{x} \cdot \cos x \)

10. \( f(x) = (5x + x^3)(6x^3 - 7x + 8) \)

11. \( f(x) = \sin x \cdot \cos x \)

12. \( f(x) = \frac{x}{\sin x} \)

13. \( f(x) = \left(\frac{3x-5}{6x+1}\right)^4 \)

14. \( f(x) = \sin^2(\cos 3x) \)

15. \( f(x) = (5x^3 + 4x)(6x - 7)^5 \)

Find \( y' \) and \( y'' \) for each of the following. Use clear notation!

16. \( y = \sin(3x^2 + 5) \)

17. \( y = \frac{x^3}{5x-4} \)

18. \( y = 5 \sin x + 10 \cos x \)

19. \( y = \frac{10}{x^{10}} \)
20. The position function of a particle is given by \( f(t) = t^3 - 15t^2 + 63t - 7 \), \( t \geq 0 \), where \( t \) is time in seconds, and \( f(t) \) is measured in feet.

   a) Find the velocity of the particle at time \( t \).

b) Find the velocity of the particle at time \( t = 5 \) seconds.

c) When is the particle at rest?

d) Find the acceleration at time \( t \).

e) Find the acceleration at time \( t = 5 \) seconds.

f) Determine intervals of time on which the particle is moving in the positive direction and in the negative direction.

\[ \frac{dy}{dx} \] by implicit differentiation. Show your work and simplify if possible.

21. \( 4x^2 - 5y^3 = 10xy \)  

22. \( x \sin y + y \sin x = 1 \)
Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

23. \(x^2 + 2xy - y^2 + x = 2\); at (1, 2)

Find \(y''\) by implicit differentiation.

24. \(x^5 + y^5 = 5\)

25. Each side of a square is increasing at a rate of 3 cm/sec. At what rate is the area of the square increasing when the side length of the square is 5 cm?

26. The radius of a sphere is increasing at a rate of 2 mm/sec. How fast is the volume increasing when the radius is 20 mm?

27. Suppose \(y = \sqrt{2x + 1}\), where \(x\) and \(y\) are functions of \(t\).

a) If \(\frac{dx}{dt} = 3\), find \(\frac{dy}{dt}\) when \(x = 4\).

b) If \(\frac{dy}{dt} = 2\), find \(\frac{dx}{dt}\) when \(x = 24\).
Sketch the derivative function for each of the functions graphed below.

28.

29.

30.

Make sure you do the suggested review problems from the book, as everything which is on the test is not covered in these 30 problems!