4.5 Optimization Problems

Steps to solving optimization problems:

1. **Understand the problem:** The first step is to read the problem carefully until it is clearly understood. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?

2. **Draw a Diagram:** In some problems it is useful to draw a diagram and identify the given and required quantities on the diagram.

3. **Introduce the Notation:** Assign a symbol to the quantity that is to be maximized or minimized (let's call it Q). Also select symbols for other unknown quantities and label the diagram with these symbols.

4. Express Q in terms of some of the other symbols from step 3.

5. If Q has been expressed as a function of more than one variable, use the given information to find relationships among the variables. Then Q will be expressed as a function of one variable. \( Q = f(x) \).

6. Use the methods from sections 4.1 and 4.3 to find the absolute maximum or minimum value of \( f \).
Find two numbers whose difference is 100 and whose product is a minimum.

The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?
A rancher is building a corral in a pasture that is adjacent to a river. The corral must have an area of 150,000 square yards. What dimensions would require the least amount of fencing if no fencing is needed for the side adjacent to the river?

What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?
Find the point on the graph of \( y = x^2 + 1 \) that is closest to the point (3, 1).

A page of a book is to have an area of 90 in\(^2\), with 1-inch margins at the bottom and sides and a 1/2-inch margin at the top. Find the dimensions of the page that will allow the largest printed area.
A huge steel cylindrical tank contains 64 m$^3$ of paint (when it is full). Find the dimensions of that cylindrical tank which will minimize the quantity of steel required to make it.

A rectangle is bounded by the $x$-axis, the $y$-axis, and the graph of $y = -2x + 7$. What dimensions of the rectangle would maximize its area?
A rancher has 10,000 feet of fencing to enclose three pens by constructing a rectangular pen with two partitions parallel to one side of the pen. What is the maximum total area of the three pens?

At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?