2.6 Implicit Differentiation

So far all of the functions that we have studied have been described by expressing one variable explicitly in terms of another variable, e.g. \( y = 3x^2 \) or \( y = x \sin(x) \). Some functions, however, are defined implicitly by a relation between \( x \) and \( y \), such as the following:

\[
\begin{align*}
\quad x^2 + y^2 &= 25 \\
\quad x^3 - y^3 &= 4xy
\end{align*}
\]

The first of these could be solved for \( y \) easily into two explicit functions, but the second one would pose some difficulties. So to differentiate an implicitly defined function, we take the derivative of both sides of the equation with respect to \( x \) and then solve the resulting equation for \( y' \). *Keep in mind that* \( y = f(x) \).

1) \( x^2 + y^2 = 25 \)  
2) \( x^3 - y^3 = 4xy \)

Find the equation of the tangent to 1) at the point (3, 4).
Find $dy/dx$ (or $y'$) by implicit differentiation for each of the following.

$$2x^3 + x^2y - xy^3 = 2$$

$$\sqrt{xy} + x - y = 7$$

$$\cos(xy) = 1 + \sin y$$

Find the equation of the tangent to the given curve at the point $(1, 2)$.

$$x^2 + 2xy - y^2 + x = 2$$
Find the equation of the tangent to the given curve at the point (0, -2).

\[ y^2 (y^2 - 4) = x^2 (x^2 - 5) \]

Find \( y'' \) by implicit differentiation.

\[ 16x^4 + y^4 = 16 \]