1.1 Functions and their Representations

Functions arise when one quantity depends on another. Although in most real-world situations there are many factors at work, we can still use the notion of a function to describe many phenomena. For example, the enrollment at ATU depends on the year; a person's weight depends on the number of calories they consume; one's distance from home depends on how long they've been traveling; the cost of mailing a package depends on its weight; the area of a circle depends on its radius; etc.

A function \( f \) is a rule that assigns to each element \( x \) in a set \( D \) exactly one element, called \( f(x) \), in a set \( E \). The set \( D \) (which includes all possible \( x \)-values for a given function) is referred to as the **domain** of the function, and the set of all possible values for \( f(x) \) is referred to as its **range**. If the function is made up of ordered pairs of the form \((x, f(x))\), \( x \) is referred to as the **independent variable** and \( f(x) \) is referred to as the **dependent variable**. It may also be helpful to think of the elements of the domain as the **inputs**, and the elements of the range as the **outputs**.

There are several different ways to think of functions, as represented in the following figures:

- A function can be represented in four different ways: verbally (in words), visually (as a graph), numerically (as a table), algebraically (with an equation or formula)

Ex: The value of a car is a function of the age of the car. As the car gets older, its value decreases. (verbal description)

Ex: \( y \) is a function of \( x \) in the following graph. (graph)

Ex: The U.S. population is a function of the year. (table)

<table>
<thead>
<tr>
<th>Year</th>
<th>U.S. Population (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>152.3</td>
</tr>
<tr>
<td>1960</td>
<td>180.7</td>
</tr>
<tr>
<td>1970</td>
<td>205.1</td>
</tr>
<tr>
<td>1980</td>
<td>227.2</td>
</tr>
<tr>
<td>1990</td>
<td>249.5</td>
</tr>
<tr>
<td>2000</td>
<td>282.2</td>
</tr>
<tr>
<td>2010</td>
<td>290.3</td>
</tr>
</tbody>
</table>

Ex: The area of a circle is a function of its radius. \( A(r) = \pi r^2 \) (equation or formula)

State the domain and range for each example above, if possible.
State the domain of each function below.

\[ f(x) = \sqrt{x + 5} \quad f(x) = \frac{4}{x^2 - x} \]

\[ f(x) = \frac{1}{\sqrt{x^2 - 9}} \]

Piecewise defined functions are functions which are defined by different formulas on different parts of their domains.

Ex:

\[ f(x) = \begin{cases} 
3 - 2x & \text{if } x \leq 0 \\
x^2 + 1 & \text{if } x > 0 
\end{cases} \]

State the domain of the piecewise function above, then evaluate it at the following, and sketch its graph.

\[ f(-2) \quad f(0) \quad f(3) \]

Write an expression for the function whose graph is described and state its domain.

The line segment joining the points (3, 4) and (1, -2).

Find a formula for the described function and state its domain.

A rectangular box with volume 10 cm³ has a square base. Express the surface area of the box as a function of the length of a side of the base.
Use the function given to do all of the following. \( f(x) = x^2 - 4x + 3 \)

a) Find \( f(5) \)

b) Find \( f(m) \)

c) Find \( f(2 + h) \)

d) Find \( \frac{f(2 + h) - f(2)}{h} \)

\[ \text{Given } f(x) = \frac{2}{x}, \text{ find the following:} \]

\[ \frac{f(3 + h) - f(3)}{h} \]
Given $f(x) = 3x^2 - 5$, find the following:

$$\frac{f(x + h) - f(x)}{h}$$

Symmetry

A function with $y$-axis symmetry is said to be an **even function**. This will only occur if $f(-x) = f(x)$.

A function with origin symmetry is said to be an **odd function**. This will only occur if $f(-x) = -f(x)$.

Determine whether each function below is even, odd, or neither.

$$f(x) = 2x^2 + 6x$$  
$$f(x) = -3x^2 + 7$$
Increasing and Decreasing Functions

A function is called **increasing** on an interval $I$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in $I$.

A function is called **decreasing** on an interval $I$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in $I$.

More simply stated, it increases if it rises from left to right and decreases if it falls from left to right.

State the interval(s) on which the function shown below is

a) increasing

b) decreasing