

6.4 Consumer and Producer Surplus

The definitions of demand and supply must be remembered: Demand tells us the price that consumers would be willing to pay for each different quantity. According to the law of demand, when the price increases the demand decreases and when the price decreases the demand increases. The graphical representation of the relationship between the quantity demanded of a good and the price of the good is known as the **demand curve**.

Supply tells us the price that producers would be willing to charge in order to sell the different quantities. The law of supply asserts that as the price of a good rises, the quantity supplied rises, and as the price of a good falls the quantity supplied falls. The graphical representation of the relationship between the quantity supplied of a good and the price of the good is known as the **supply curve**.

The demand and supply curve intersects at the **point of equilibrium** (q^*, p^*) . We call p^* the **equilibrium price** and the q^* the **equilibrium quantity**. See Figure 6.4.1.

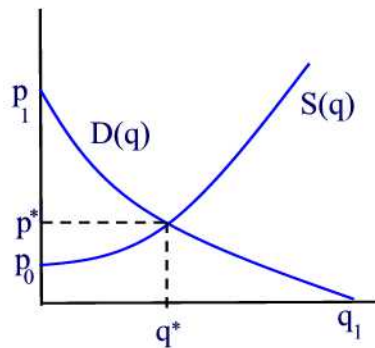


Figure 6.4.1

Two important applications of the demand and supply curves in economics: The consumer's surplus and the producer's surplus.

Consumers' Surplus

Consumers' surplus is the economic gain accruing to a consumer (or consumers) when they engage in trade. The gain is the difference between the price they are willing to pay and the actual price.

At the equilibrium level, the consumers' surplus is the difference between

what consumers are willing to pay and their actual expenditure: It therefore represents the total amount saved by consumers who were willing to pay more than p^* per unit.

To calculate the consumers' surplus, we first calculate the consumers' total expenditure. Divide the interval $[0, q^*]$ into n equal pieces each of length Δq . According to Figure 6.4.2, the first quantity Δq is sold at the price $D(q_1)$, the second quantity Δq is sold at $D(q_2)$, and the last quantity Δq is sold at a price of $D(q_n)$. Hence, the consumers' total expenditure is given by the sum

$$D(q_1)\Delta q + D(q_2)\Delta q + \cdots + D(q_n)\Delta q = \sum_{i=1}^n D(q_i)\Delta q.$$

Letting $\Delta q \rightarrow 0$ to obtain (See Figure 6.4.2)

$$\text{Total Expenditure} = \int_0^{q^*} D(q) dq.$$

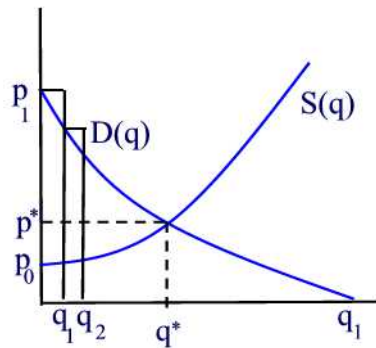


Figure 6.4.2

Thus,

$$\text{Consumers' Surplus} = \int_0^{q^*} D(q) dq - p^* q^*.$$

Note that $p^* q^*$ is the actual expenditure if the goods are sold at the equilibrium price. Graphically, the consumers' surplus is the area between demand curve and the horizontal line at p^* . See Figure 6.4.3.

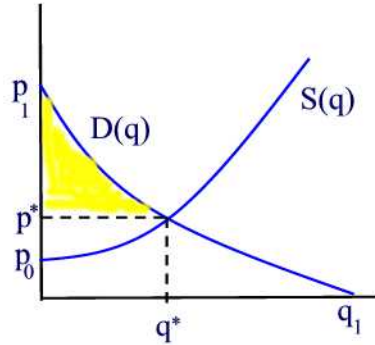


Figure 6.4.3

Producers' Surplus

The producer surplus measures the suppliers' gain from trade. It is the total amount gained by producers by selling at the current price, rather than at the price they would have been willing to accept. For example, a producer might be willing to sell at a price lower than p^* so that he can stay in business. However, by selling at the price of p^* he is making a benefit which is the producers' surplus. Thus, the producers' surplus can be defined as the extra amount earned by producers who were willing to charge less than the selling price of p^* per unit, and is given by

$$\text{Producers' Surplus} = p^* q^* - \int_0^{q^*} S(q) dq.$$

Graphically, it is the area between supply curve and the horizontal line at p^* . See Figure 6.4.4.

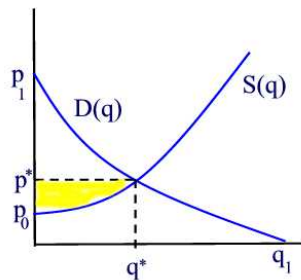


Figure 6.4.4

Example 6.4.1

The demand and supply equations are given by $D(q) = 60 - \frac{q^2}{10}$ and $S(q) = 30 + \frac{q^2}{5}$. Find the consumers' and producers' surpluses at the equilibrium price.

Solution.

To find the Consumers' and Producers' surpluses under equilibrium we first need to find the equilibrium point by setting supply = demand and solving for q :

$$30 + \frac{q^2}{5} = 60 - \frac{q^2}{10} \text{ implies } q^* = 10.$$

Substituting this into the supply (or demand) equation we find the equilibrium price $p^* = 50$. Now we use formulas of the Consumers' and Producers' surpluses:

$$\text{Consumers' Surplus : } \int_0^{10} \left[\left(60 - \frac{q^2}{10} \right) - 50 \right] dq = 10q - \frac{q^3}{30} \Big|_0^{10} \approx 66.67.$$

This tells us that consumers gain \$66.67 in buying goods at the equilibrium price instead of a price they would have been willing to pay and which is usually higher than the equilibrium price.

$$\text{Producers' Surplus : } \int_0^{10} \left[50 - \left(30 + \frac{q^2}{5} \right) \right] dq = 20q - \frac{q^3}{15} \Big|_0^{10} \approx 133.33.$$

This tells us that producers gain \$133.33 by supplying goods at the equilibrium price instead of a price at which they are willing to provide for the goods which is usually a price lower than the equilibrium price ■