4.6 Elasticity of Demand

An important quantity in economics theory is the **price elasticity of demand** which measures the responsiveness of demand to a given change in price and is found using the formula

$$\begin{aligned} E &= \left| \frac{\text{percentage change in quantity demanded}}{\text{percentage change in price}} \right| \\ &= \left| \frac{\frac{dq}{q}}{\frac{dp}{p}} \right| \\ &= \left| \frac{p}{q} \cdot \frac{dq}{dp} \right|. \end{aligned}$$

We will assume that increasing the price usually decreases demand and decreasing the price will increase demand so that $\frac{dq}{q}$ and $\frac{dp}{p}$ have opposite sign, that is, their ratio is always negative. Thus,

$$\frac{\Delta q}{q} \approx -E \frac{\Delta p}{p}.$$

Changing the price of an item by 1% causes a change of E% in the quantity sold.
If $E > 1$ then this means that an increase (or decrease) of 1% in price causes demand to drop (increase) by more than one percent. In this case, we say that the demand is **elastic.** If $0 \leq E < 1$ then an increase (decrease) of 1% in price causes demand to drop (increase) by less than one percent and in this case we say that the demand is **inelastic**.

Example 4.6.1
Raising the price of hotel rooms from \$75 to \$80 per night reduces weekly sales from 100 rooms to 90 rooms.

(a) What is the elasticity of demand for rooms at a price of \$75?
(b) Should the owner raise the price?

Solution.
(a) The percent change in price is $\frac{\Delta p}{p} = \frac{80-75}{75} = 0.067 = 6.7\%$ and the percent

change in demand is $\frac{\Delta q}{q} = \frac{90-100}{100} = -0.1 = -10\%$. Thus, the elasticity of demand is $E = \frac{0.1}{0.067} = 1.5$.
(b) The weekly revenue at the price of \$75 is $100 \cdot 75 = \$7500$ whereas at the price of \$80 the weekly revenue is $90 \cdot 80 = \$7200$. A price increase results in loss of revenue, so the price should not be raised ■

Example 4.6.2
The demand for a product is $q = 2000 - 5p$ where q is units sold at a price of p dollars. Find the elasticity if the price is \$10, and interpret your answer in terms of demand.

Solution.
Using Leibniz notation we find $\left.\frac{dq}{dp}\right|_{p=10} = -5$ and for $p = 10$ the corresponding quantity is $q = 2000 - 50 = 1950$ so that the elasticity is

$$E = \left|\frac{p}{q}\frac{dq}{dp}\right| = \frac{10 \cdot 5}{1950} = 0.03.$$

The demand is inelastic at the given price; a 1% increase in price will result in a decrease of 0.03% in demand ■
Finally, we would like to know the price that maximizes revenue. That is, the price that brings the greatest revenue. Recall that the revenue function is given by $R = pq$ so that $\frac{dR}{dp} = q + p\frac{dq}{dp} = q(1 + \frac{p}{q}\frac{dq}{dp})$.
If $E > 1$ then $\frac{p}{q}\frac{dq}{dp} < -1$ so that $1 + \frac{p}{q}\frac{dq}{dp} < 0$ and therefore $\frac{dR}{dp} < 0$. This says, that increasing price will decrease revenue or decreasing the price will increase revenue. If $E < 1$ then $\frac{p}{q}\frac{dq}{dp} > -1$ so that $1 + \frac{p}{q}\frac{dq}{dp} > 0$ and consequently $\frac{dR}{dp} > 0$. This means that increasing price will increase revenue. Finally, note that if $\frac{dR}{dp} = 0$ then $E = 1$. That is, $E = 1$ at the critical points of the revenue function. See Figure 4.6.1.

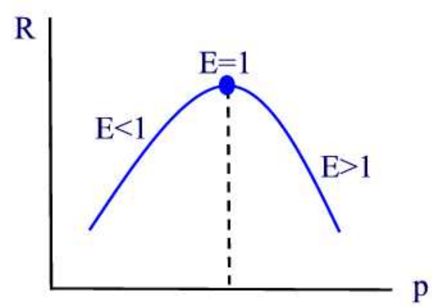

Figure 4.6.1