

4.5 Average Cost

We have seen that an important principle in economics is the problem of maximizing profit. A second general principle involves the relationship between the marginal cost and the **average cost**

$$a(q) = \frac{C(q)}{q}, \quad q > 0.$$

That is, the average cost is the cost per unit of producing a certain quantity. It is important to notice that the marginal cost (the cost of producing the next item) and average cost do not mean the same thing as the following example shows.

Example 4.5.1

The cost of producing q items is $C(q) = 2500 + 12q$ dollars.

- What is the marginal cost of producing the 100th item?
- What is the average cost of producing 100 items?

Solution.

(a) The marginal cost at the level q is given by $MC(q) = C'(q) = 12$ dollars per unit. This means that after producing the 99 items, it costs an additional \$12 to produce the 100th item.

(b) $a(100) = \frac{C(100)}{100} = \frac{2500+12(100)}{100} = \37 per item ■

Since $a(q) = \frac{C(q)}{q} = \frac{C(q)-0}{q-0}$, $a(q)$ is the slope of the line passing through the points $(q, C(q))$ and the origin $(0, 0)$. See Figure 4.5.1.

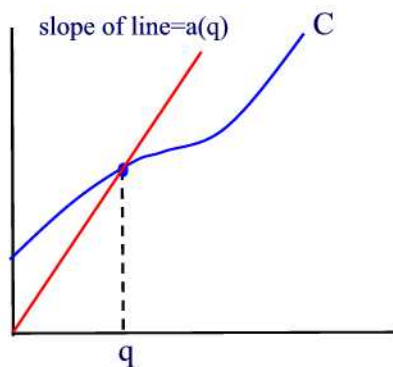


Figure 4.5.1

Minimizing $a(q)$

The important question in this section is the question of minimizing the average cost function $a(q)$. Let's try to find the derivative of $a(q)$. Using the quotient rule of differentiation we obtain

$$a'(q) = \frac{C''(q)q - C'(q)}{q^2} = \frac{C''(q) - a(q)}{q}.$$

Thus, $a'(q) = 0$ when $C''(q) = a(q)$. So critical numbers of $a(q)$ satisfy the relationship $C''(q) = a(q)$. In economics theory the global minimum of $a(q)$ occurs at a critical number of $a(q)$. Graphically, the minimum average cost occurs at the point on the graph of $C(q)$ where the line passing through the origin is tangent to the graph of $C(q)$. See Figure 4.5.2.

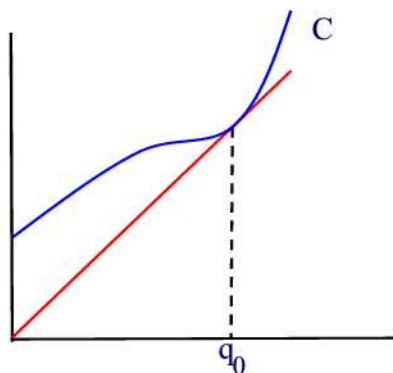


Figure 4.5.2

Thus, if q_0 is a critical number of $a(q)$ then for $q < q_0$ the marginal cost is less than the average cost, i.e., $a'(q) < 0$. This means, increasing production will decrease the average cost. If, on the other hand, $q > q_0$ then the marginal cost is greater than the average cost, i.e., $a'(q) > 0$. This means that increasing production will increase the average cost.

Example 4.5.2

A total cost function, in thousands of dollars, is given by $C(q) = q^3 - 6q^2 + 15q$, where q is in thousands and $0 \leq q \leq 5$.

- Graph $C(q)$. Estimate the quantity at which average cost is minimized.
- Graph the average cost function. Use it to estimate the minimum average cost.

(c) Determine analytically the exact value of q at which average cost is minimized.

Solution.

(a) A graph of $C(q)$ is given in Figure 4.5.3. The average cost is minimized at the point where the line going through the origin is tangent to the graph of $C(q)$. This occurs at approximately $q = 3$.

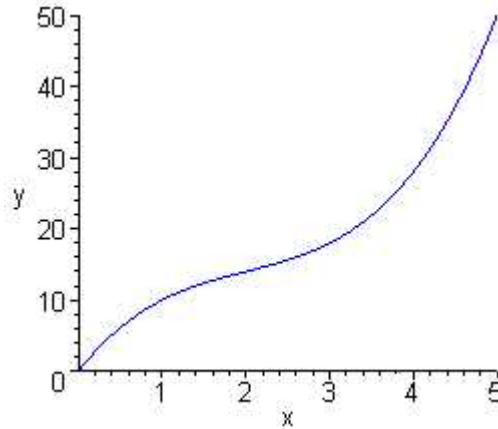


Figure 4.5.3

(b) The average cost function is given by $a(q) = \frac{C(q)}{q} = q^2 - 6q + 15$. The graph of this function is given in Figure 4.5.4. Notice that the minimum occurs at approximately $q = 3$.

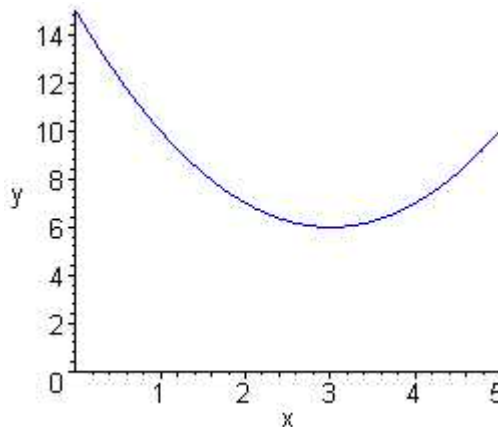


Figure 4.5.4

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(c) The minimum average cost occurs when $C'(q) = a(q)$. That is, $3q^2 - 12q + 15 = q^2 - 6q + 15$. This gives $2q^2 - 6q = 0$. Solving for q we find $q = 0$ or $q = 3$. Since the average cost is not defined when $q = 0$, the average cost is minimum at $q = 3$ ■