

4.3 Global Maxima and Minima

In this section we will look for the largest or the smallest values of a function on its domain. Such points are called **global extrema**. If $f(a)$ is the largest value then it satisfies the inequality $f(x) \leq f(a)$ for all x in the domain of f . We call $f(a)$ the **global or absolute maximum value** of f and the point $(a, f(a))$ the global maximum point. Similarly, if $f(a)$ is the smallest value of $f(x)$ then $f(a) \leq f(x)$ for all x in the domain of f . We call $f(a)$ the **absolute or global minimum value** of f and the point $(a, f(a))$ the global minimum.

The process of finding the global extrema is called **optimization**. Problems that involve finding the global extrema are called **optimization problems**.

How do we find the global extrema?

- If the function is continuous on a closed interval then the global extrema occur at either the critical points or the endpoints of the interval.

Example 4.3.1

Find the global extrema of the function $f(x) = x^3 - 9x^2 - 48x + 52$ on the closed interval $[-5, 12]$.

Solution.

Finding the derivative of $f(x)$ we get $f'(x) = 3x^2 - 18x - 48$. Solving the equation $f'(x) = 0$ that is, $x^2 - 6x - 16 = 0$ we find the critical points at $x = 8$ and $x = -2$. Now, evaluating the function at these points and at the endpoints we find

$$\begin{aligned}f(-5) &= -58 \\f(-2) &= 104 \\f(8) &= -396 \\f(12) &= -92\end{aligned}$$

It follows that $(-2, 104)$ is the global maximum point and $(8, -396)$ is the global minimum point ■

- If a function is continuous on an open interval or on all real numbers then it is recommended to find the global extrema by graphing the function.

Example 4.3.2

Find the global extrema of the function $f(x) = 100(e^{-0.02x} - e^{-0.1x})$ for $x \geq 0$.

Solution.

Let's sketch the graph of this function. The standard process of graphing consists of the following steps:

Step 1. Find the critical numbers. Setting $f'(x) = 0$ to obtain

$$\begin{aligned}
 100(-0.02e^{-0.02x} + 0.1e^{-0.1x}) &= 0 \\
 0.02e^{-0.02x} &= 0.1e^{-0.1x} \\
 \frac{e^{-0.02x}}{e^{-0.1x}} &= \frac{0.1}{0.02} \\
 e^{0.08x} &= 5 \\
 0.08x &= \ln 5 \\
 x &= \frac{\ln 5}{0.08} = 20.12
 \end{aligned}$$

Step 2. We construct the following chart:

x		20.12	
f'(x)	+	0	-
f(x)	↗	53.50	↘

Step 3. Find the second derivative to obtain $f''(x) = 100(0.0004e^{-0.02x} - 0.01e^{-0.1x})$. Setting this to zero and solving for x as in Step 1 we find $x \approx 40.25$. Now we construct the table

x		40.25	
f''(x)	-	0	+
f(x)	∩	f(40.25)	∪

Step 4. Graph

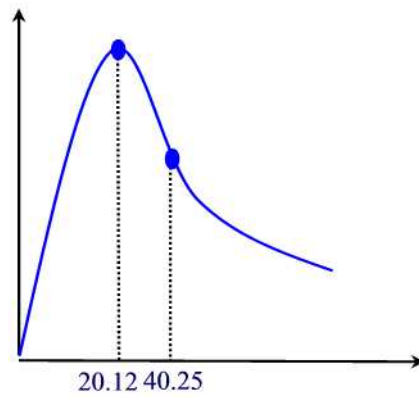


Figure 4.3.1

Thus, from the graph we see that $(20.12, 53.50)$ is a global maximum. The function has a global minimum at $x = 0$. ■