2.5 Marginal Cost and Revenue

We start this section by looking at possible graphs of the cost and revenue functions.
A cost function can be linear as shown in Figure 2.5.1(a) , or have the shape shown in Figure 2.5.1(b). Note that in Figure 2.5.1(b), the graph is concave down then concave up. This means that the cost function increases first at a slow rate, then slows down, and finally increases at a faster rate.

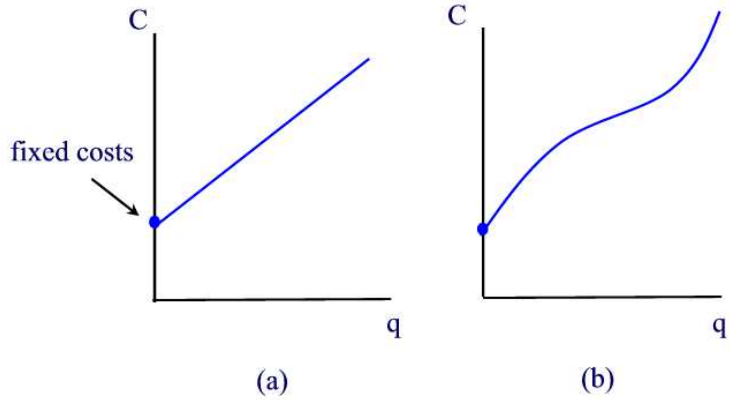

Figure 2.5.1

Now, since $R = pq$, the graph of R as a function of q is a straight line going through the origin and with slope p when the price p is constant (See Figure 2.5.2(a)), or the graph shown in Figure 2.5.2(b).

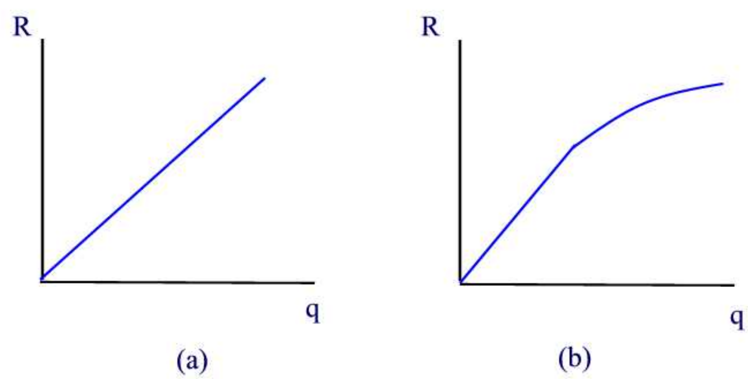

Figure 2.5.2

Marginal Analysis
Marginal analysis is an area of economics concerned with estimating the effect on quantities such as cost, revenue, and profit when the level of production is changed by a unit amount. For example, if $C(q)$ is the cost of producing q units of a certain commodity, then the **marginal cost**, $MC(q)$, is the additional cost of producing one more unit and is given by the difference $MC(q) = C(q+1) - C(q)$. Using the estimation

$$C'(q) \approx \frac{C(q+1) - C(q)}{(q+1) - q} = C(q+1) - C(q)$$

we find that

$$MC(q) \approx C'(q)$$

and for this reason, we will compute the marginal cost by the derivative $C'(q)$.
Similarly, if $R(q)$ is the revenue obtained from producing q units of a commodity, then the **marginal revenue**, $MR(q)$, is the additional revenue obtained from producing one more unit, and we compute $MR(q)$ by the derivative $R'(q)$.

Example 2.5.1
Let $C(q)$ represent the cost, $R(q)$ the revenue, and $P(q)$ the total profit, in dollars, of producing q units.

(a) If $C'(50) = 75$ and $R'(50) = 84$, approximately how much profit is earned by the 51^{st}item?
(b) If $C'(90) = 71$ and $R'(90) = 68$, approximately how much profit is earned by the 91^{st}item?

Solution.
(a) $P'(50) = R'(50) - C'(50) = 84 - 75 = 9$.
(b) $P'(90) = R'(90) - C'(90) = 68 - 71 = -3$. A loss by 3 dollars ■

Example 2.5.2
Cost and Revenue are given in Figure 2.5.3. Sketch the graphs of the marginal

cost and marginal revenue.

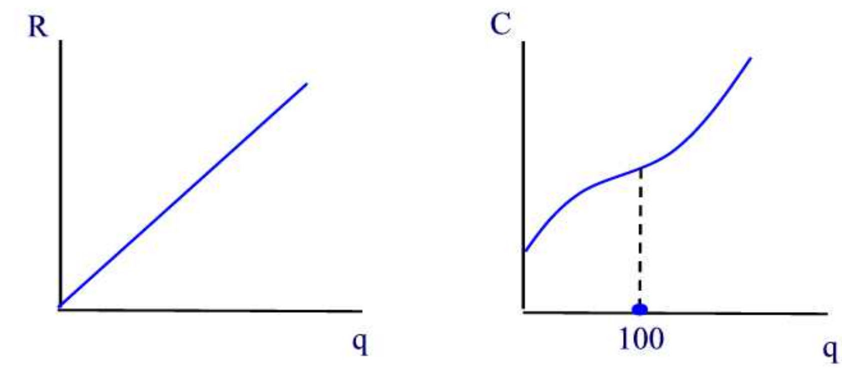

Figure 2.5.3

Solution.
Since the graph of R is a straight line with positive slope p, the graph of MR is a horizontal line at p. (See Figure 2.5.4(a)). For the marginal cost, note that the marginal cost is decreasing for $q < 100$ and then increasing for $q > 100$. Thus, $q = 100$ is a minimum point. (See Figure 2.5.4(b)) ■

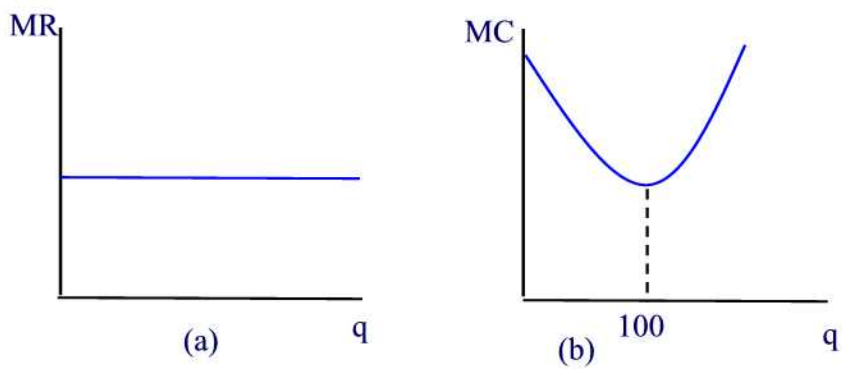

Figure 2.5.4

Maximizing Profit

We end this section by considering the question of maximizing the profit function P. That is, maximizing the function

$$P(q) = R(q) - C(q).$$

We will see in Section 4.1, that the profit function attains its maximum for the level of production q for which $P'(q) = 0$, i.e., $MC(q) = MR(q)$. Geometrically, this occurs at q where the tangent line to the graph of C is parallel to the tangent line to the graph of R at q.

Example 2.5.3
A manufacturer estimates that when q units of a particular commodity are produced each month, the total cost (in dollars) will be

$$C(q) = \frac{1}{8}q^2 + 4q + 200$$

and all units are sold at a price $p = 49 - q$ dollars per unit. Determine the price that corresponds to the maximum profit.

Solution.
The revenue function is given by

$$R(q) = pq = 49q - q^2$$

and its derivative is $MR(q) = 49 - 2q$. Setting this expression equal to the marginal cost to obtain

$$\frac{1}{4}q + 4 = 49 - 2q.$$

Solving for q we obtain $q = 20$ units. Thus, $p = 49 - 20 = \$29$ ∎

Example 2.5.4
Locate the quantity in Figure 2.5.5 where the profit function is maximum.

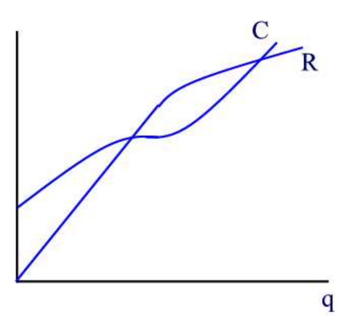

Figure 2.5.5

Solution.
The quantity q' for which profit is maximized is shown in Figure 2.5.6 where the tangent line to the graph of C at q' is parallel to the tangent line to the graph of R at q' ■

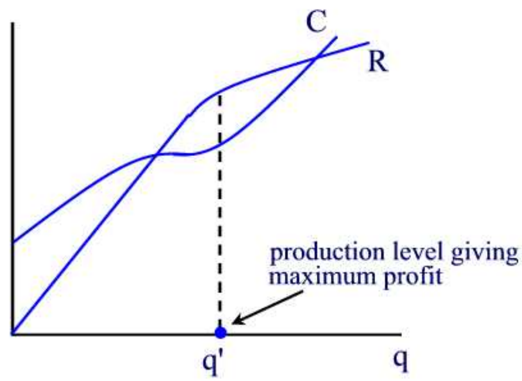

Figure 2.5.6