Section 6.3

1. Since $F'(x) = 6x$, we use $F(x) = 3x^2$. By the Fundamental Theorem, we have

$$\int_0^4 6x dx = 3x^2\Big|_0^4 = 3 \cdot 4^2 - 3 \cdot 0^2 = 48 - 0 = 48.$$

7. Since $F'(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$, we use $F(x) = 2x^{1/2} = 2\sqrt{x}$. By the Fundamental Theorem, we have

$$\int_1^4 \frac{1}{\sqrt{x}} dx = 2\sqrt{x}\Big|_1^4 = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = 2.$$

13. Since $F'(y) = y^2 + y^4$, we take $F(y) = \frac{y^3}{3} + \frac{y^5}{5}$. Then

$$\begin{aligned}\int_0^1 (y^2 + y^4)\, dy &= F(3) - F(0) \\ &= \left(\frac{1^3}{3} + \frac{1^5}{5}\right) - \left(\frac{0^3}{3} + \frac{0^5}{5}\right) \\ &= \frac{1}{3} + \frac{1}{5} = \frac{8}{15}.\end{aligned}$$

19.

If $f(t) = e^{0.05t}$, then $F(t) = 20e^{0.05t}$ (you can check this by observing that $\frac{d}{dt}(20e^{0.05t}) = e^{0.05t}$). By the Fundamental Theorem, we have

$$\int e^{0.05t}\, dt = 20e^{0.05t}\Big|_0^3 = 20e^{0.15} - 20e^0 = 20(e^{0.15} - 1).$$

23. We have

$$\text{Area} = \int_0^b x^2 dx = \frac{x^3}{3}\Big|_0^b = \frac{b^3}{3}.$$

We find the value of b making the area equal to 100:

$$\begin{aligned}100 &= \frac{b^3}{3} \\ 300 &= b^3 \\ b &= (300)^{1/3} = 6.694.\end{aligned}$$

26. **(a)** At time $t = 0$, the rate of oil leakage $= r(0) = 50$ thousand liters/minute.
At $t = 60$, rate $= r(60) = 15.06$ thousand liters/minute.
(b) To find the amount of oil leaked during the first hour, we integrate the rate from $t = 0$ to $t = 60$:

$$\text{Oil leaked} = \int_0^{60} 50e^{-0.02t}\, dt = \left(-\frac{50}{0.02}e^{-0.02t}\right)\Bigg|_0^{60}$$
$$= -2500e^{-1.2} + 2500e^0 = 1747 \text{ thousand liters.}$$

30.

(a) The graph of $y = e^{-x^2}$ is in Figure 7.4. The integral $\int_{-\infty}^{\infty} e^{-x^2}\, dx$ represents the entire area under the curve, which is shaded.

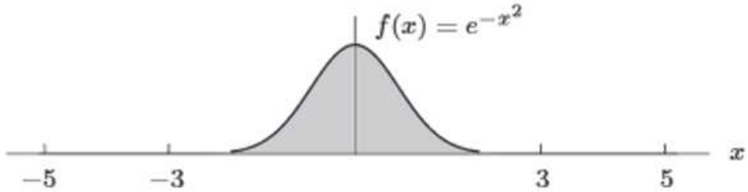

Figure 7.4

(b) Using a calculator or computer, we see that

$$\int_{-1}^{1} e^{-x^2}\, dx = 1.494, \quad \int_{-2}^{2} e^{-x^2}\, dx = 1.764, \quad \int_{-3}^{3} e^{-x^2}\, dx = 1.772, \quad \int_{-5}^{5} e^{-x^2}\, dx = 1.772$$

(c) From part (b), we see that as we extend the limits of integration, the area appears to get closer and closer to about 1.772. We estimate that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = 1.772$$

33.

(a) The total number of people that get sick is the integral of the rate. The epidemic starts at $t = 0$. Since the rate is positive for all t, we use ∞ for the upper limit of integration.

$$\text{Total number getting sick} = \int_0^{\infty} \left(1000te^{-0.5t}\right)\, dt$$

(b) The graph of $r = 1000te^{-0.5t}$ is shown in Figure 7.6. The shaded area represents the total number of people who get sick.

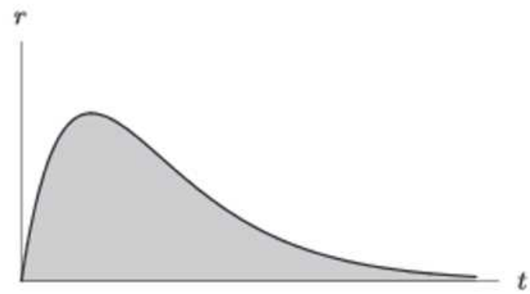

Figure 7.6