# Section 5.6

1.

**(a)** Since $f(x)$ is positive on the interval from 0 to 6, the integral is equal to the area under the curve. By examining the graph, we can measure and see that the area under the curve is 20 square units, so

$$\int_0^6 f(x)dx = 20.$$

**(b)** The average value of $f(x)$ on the interval from 0 to 6 equals the definite integral we calculated in part (a) divided by the size of the interval. Thus

$$\text{Average Value} = \frac{1}{6}\int_0^6 f(x)dx = 3\frac{1}{3}.$$

3. $$\text{Average value} = \frac{1}{2-0}\int_0^2 (1+t)\,dt = \frac{1}{2}(4) = 2.$$

6. By a visual estimate, the average value is $\approx -3$.

9. It appears that the area under a line at about $y = 17$ is approximately the same as the area under $f(x)$ on the interval $x = a$ to $x = b$, so we estimate that the average value is about 17.

11. **(a)** The average inventory is given by the formula

$$\frac{1}{90-0}\int_0^{90} 5000(0.9)^t dt.$$

Using a calculator yields

$$\int_0^{90} 5000(0.9)^t dt \approx 47452.5$$

so the average inventory is

$$\frac{47452.5}{90} \approx 527.25.$$

**(b)** The function is graphed in Figure 6.3. The area of a rectangle of height 527.25 is equal to the area under the curve.

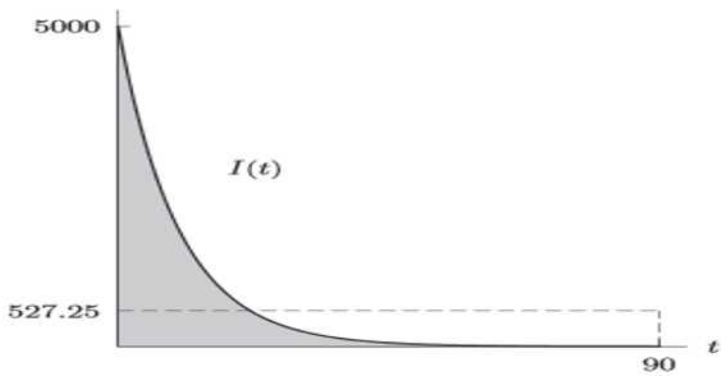



**Figure 6.3**

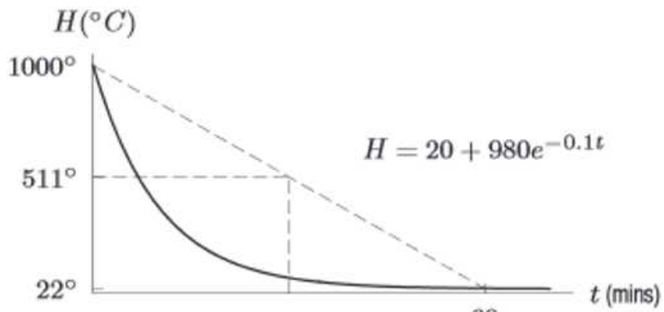
19.

**(a)** At the end of one hour $t = 60$, and $H = 22^\circ$C.

**(b)**

$$\text{Average temperature} = \frac{1}{60}\int_0^{60} (20 + 980e^{-0.1t})dt$$
$$= \frac{1}{60}(10976) = 183^\circ\text{C}.$$

**(c)** Average temperature at beginning and end of hour $= (1000 + 22)/2 = 511^\circ$C. The average found in part (b) is smaller than the average of these two temperatures because the bar cools quickly at first and so spends less time at high temperatures. Alternatively, the graph of $H$ against $t$ is concave up.




20. **(a)** See Figure 6.7. More sales were made in the second half of the year, because the area under the second half of the curve is greater.

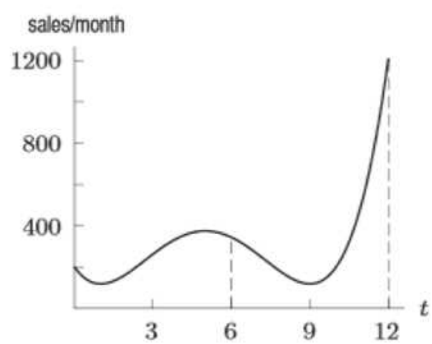



**Figure 6.7**

**(b)** The total sales for the first six months amount to

$$\int_0^6 r(t)dt = \$1531.20$$

The total sales for the last 6 months of the year amount to

$$\int_6^{12} r(t)dt = \$1963.20$$

**(c)** Thus the total sales for the year amount to

$$\int_0^{12} r(t)dt = \int_0^{6} r(t)dt + \int_6^{12} r(t)dt = \$1531.20 + \$1963.20 = \$3494.40$$

**(d)** The average sales per month is the quotient of the total sales with 12 months giving

$$\frac{\text{Total sales}}{\text{12 months}} = \frac{3494.4}{12} = \$291.20\text{/month.}$$

# 23.

In (a), $f'(1)$ is the slope of a tangent line at $x = 1$, which is negative. As for (c), the rate of change in $f(x)$ is given by $f'(x)$, and the average value of this over $0 \le x \le a$ is

$$\frac{1}{a-0}\int_0^a f'(x)\,dx = \frac{f(a)-f(0)}{a-0}.$$

This is the slope of the line through the points $(0,1)$ and $(a,0)$, which is less negative that the tangent line at $x = 1$. Therefore, $(a) < (c) < 0$. The quantity (b) is $\left(\int_0^a f(x)\,dx\right)/a$ and (d) is $\int_0^a f(x)\,dx$, which is the net area under the graph of $f$ (counting the area as negative for $f$ below the $x$-axis). Since $a > 1$ and $\int_0^a f(x)\,dx > 0$, we have 0 <(b)<(d). Therefore

$$\text{(a)} < \text{(c)} < \text{(b)} < \text{(d)}.$$