Section 5.5

1. If $H(t)$ is the temperature of the coffee at time t, by the Fundamental Theorem of Calculus

$$\text{Change in temperature} = H(10) - H(0) = \int_0^{10} H'(t)\,dt = \int_0^{10} -7(0.9^t)\,dt.$$

Therefore,

$$H(10) = H(0) + \int_0^{10} -7(0.9^t)\,dt \approx 90 - 44.2 = 45.8^\circ\text{C}.$$

3. The fixed cost is $C(0) = 1{,}000{,}000$.

$$\text{Total variable cost} = \int_0^{500} C'(x)\,dx = \int_0^{500} (4000 + 10x)\,dx = 3{,}250{,}000.$$

Therefore,

$$\begin{aligned}\text{Total cost} &= \text{Fixed Cost} + \text{Total variable cost}\\ &= 4{,}250{,}000 \text{ riyals.}\end{aligned}$$

5.

(a) There are approximately 5.5 squares under the curve of $C'(q)$ from 0 to 30. Each square represents \$100, so the total variable cost to produce 30 units is around \$550. To find the total cost, we had the fixed cost

$$\begin{aligned}\text{Total cost} &= \text{fixed cost} + \text{total variable cost}\\ &= 10,000 + 550 = \$10,550.\end{aligned}$$

(b) There are approximately 1.5 squares under the curve of $C'(q)$ from 30 to 40. Each square represents \$100, so the additional cost of producing items 31 through 40 is around \$150.

(c) Examination of the graph tells us that $C'(25) = 10$. This means that the cost of producing the 26th item is approximately \$10.

7. **(a)**

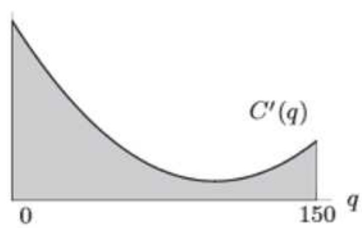

Figure 5.51

The total variable cost of producing 150 units is represented by the area under the graph of $C'(q)$ between 0 and 150, or

$$\int_0^{150} (0.005q^2 - q + 56)dq.$$

(b) An estimate of the total cost of producing 150 units is given by

$$20{,}000 + \int_0^{150} (0.005q^2 - q + 56)dq.$$

This represents the fixed cost ($20,000) plus the variable cost of producing 150 units, which is represented by the integral. Using a calculator, we see

$$\int_0^{150} (0.005q^2 - q + 56)dq \approx 2{,}775.$$

So the total cost is approximately

$$\$20{,}000 + \$2{,}775 = \$22{,}775.$$

(c) $C'(150) = 0.005(150)^2 - 150 + 56 = 18.5$. This means that the marginal cost of the 150th item is 18.5. In other words, the 151st item will cost approximately $18.50.

(d) $C(151)$ is the total cost of producing 151 items. This can be found by adding the total cost of producing 150 items (found in part (b)) and the additional cost of producing the 151st item ($C'(150)$, found in (c)). So we have

$$C(151) \approx 22{,}775 + 18.50 = \$22{,}793.50.$$

8. **(a)** Total variable cost in producing 400 units is

$$\int_0^{400} C'(q)\,dq.$$

We estimate this integral:

$$\begin{aligned} \text{Left–hand sum } &= 25(100) + 20(100) + 18(100) + 22(100) = 8500; \\ \text{Right–hand sum } &= 20(100) + 18(100) + 22(100) + 28(100) = 8800; \end{aligned}$$

and so $\int_0^{400} C'(q)\,dq \approx \dfrac{8500 + 8800}{2} = \$8650.$

$$\begin{aligned} \text{Total cost} &= \text{ Fixed cost } + \text{ Variable cost} \\ &= \$10{,}000 + \$8650 = \$18{,}650. \end{aligned}$$

(b) $C'(400) = 28$, so we would expect that the 401st unit would cost an extra $28.

9. Since $C(0) = 500$, the fixed cost must be $500. The total *variable* cost to produce 20 units is

$$\int_0^{20} C'(q)\,dq = \int_0^{20} (q^2 - 16q + 70)\,dq = \$866.67 \text{ (using a calculator).}$$

The *total* cost to produce 20 units is the fixed cost plus the variable cost of producing 20 units. Thus,

$$\text{Total cost} = \$500 + \$866.67 = \$1{,}366.67.$$