Section 5.4

1. The integral $\int_1^3 v(t)\,dt$ represents the change in position between time $t = 1$ and $t = 3$ seconds; it is measured in meters.

5.

For any t, consider the interval $[t, t+\Delta t]$. During this interval, oil is leaking out at an approximately constant rate of $f(t)$ gallons/minute. Thus, the amount of oil which has leaked out during this interval can be expressed as

$$\text{Amount of oil leaked} = \text{Rate} \times \text{Time} = f(t)\,\Delta t$$

and the units of $f(t)\,\Delta t$ are gallons/minute $\times$ minutes $=$ gallons. The total amount of oil leaked is obtained by adding all these amounts between $t = 0$ and $t = 60$. (An hour is 60 minutes.) The sum of all these infinitesimal amounts is the integral

$$\begin{matrix}\text{Total amount of} \\ \text{oil leaked, in gallons}\end{matrix} = \int_0^{60} f(t)\,dt.$$

9.

(a) In 1990, when $t = 0$, gas consumption was 1770 millions of metric tons of oil equivalent. In 2010, when $t = 20$, gas consumption was $N = 1770 + 53(20) = 2830$ million metric tons of oil equivalent.

(b) We use an integral to approximate the sum giving the total amount consumed over the 20-year period:

$$\text{Total amount of gas consumed} = \int_0^{20} (1770 + 53t)\,dt = 46{,}000 \text{ million metric tons of oil equivalent.}$$

19.

(a) Since velocity is the rate of change of distance, we have

$$\text{Distance traveled} = \int_0^5 (10 + 8t - t^2)\,dt.$$

This distance is the shaded area in Figure 5.59.

(b) A graph of this velocity function is given in Figure 5.59. Finding the distance traveled is equivalent to finding the area under this curve between $t = 0$ and $t = 5$. We estimate that this area is about 100 since the average height appears to be about 20 and the width is 5.

(c) We use a calculator or computer to calculate the definite integral

$$\text{Distance traveled} = \int_0^5 (10 + 8t - t^2)\,dt \approx 108.33 \quad \text{meters.}$$

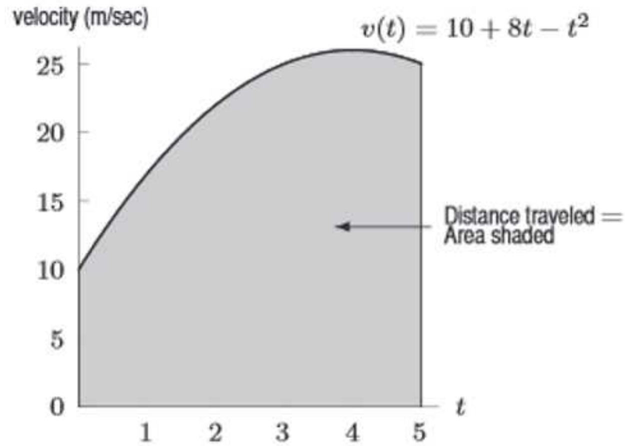

Figure 5.59: A velocity function

25.

(a) (i) The income curve shows the rate of change of the value of the fund due to inflow of money. The area under the curve,

$$\int_{2000}^{2015} I(t)\,dt,$$

represents the total change in the value of the fund that is due to income. It is the quantity of money, in billions of dollars, that is projected to flow into the fund between 2000 and 2015.

(ii) The expenditure curve shows the rate of change of the value of the fund due to outflow of money. The area under the curve,

$$\int_{2000}^{2015} E(t)\,dt,$$

represents the magnitude of the change in the value of the fund that is due to expenses. It is the quantity of money, in billions of dollars, that is projected to flow out of the fund between 2000 and 2015.

(iii) The area between the income and expenditure curves,

$$\int_{2000}^{2015} I(t) - E(t)\,dt,$$

represents the difference between total income and total expenses between 2000 and 2015. It is the projected change in value of the fund between 2000 and 2015.

(b) In the figure, we see that the value of the fund was about 1000 billion dollars in 2000 and is projected to be about 3500 billion dollars in 2015. The fund is projected to increase in value by about 2500 billion dollars, and that is the area between the income and expenditure curves on the graph.

32.

(a) Figure 5.47 is the graph of a rate of blood flow versus time. The total quantity of blood pumped during the three hours is given by the area under the rate graph for the three-hour time interval. The area can be estimated by counting grid boxes under the graph.

Each grid rectangle has area 30 minutes $\times 1/2$ liter/minute $= 15$ liters, representing 15 liters of blood pumped. The grid boxes in the graph are stacked in six columns. Estimating the number of boxes in each column under the graph gives

$$\text{Number of boxes} = 10 + 7 + 3.75 + 3.5 + 3 + 1.5 = 28.75 \text{ boxes.}$$

Approximately

$$\text{Amount of blood pumped} = (28.75)(15) = 431.25 \text{ liters.}$$

Thus, about 431 liters of blood are pumped during the three hours leading to death.

(b) Since $f(t)$ is the pumping rate in liters/minute at time t hours, $60f(t)$ is the pumping rate in liters/hour. Thus $\int_0^3 60f(t)\,dt$ gives the total quantity of blood pumped in liters during the three hours.

(c) During three hours with no bleeding, the heart pumps 5 liters/minute for $3 \cdot 60 = 180$ minutes. Thus

$$\text{Total blood pumped} = 5 \cdot 180 = 900 \text{ liters.}$$

This is $900 - 431.25 = 468.75$ liters more than pumped with 2 liter bleeding. Thus, about 470 liters are pumped. This corresponds to the area on the graph between the 5 liters/minute line and the pumping rate for the 2 liter bleed. See Figure 5.48.

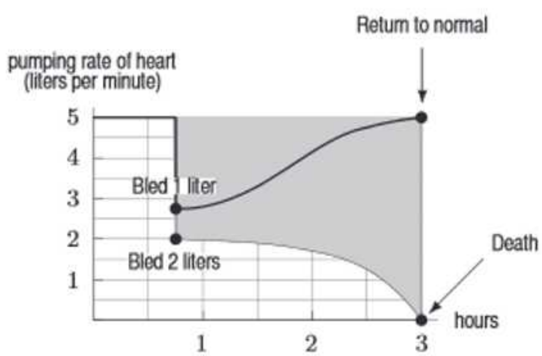

Figure 5.47

34.

Since W is in tons per week and t is in weeks since January 1, 2005, the integral $\int_0^{52} W\,dt$ gives the amount of waste, in tons, produced during the year 2005.

$$\text{Total waste during the year} = \int_0^{52} 3.75e^{-0.008t}\,dt = 159.5249 \text{ tons.}$$

Since waste removal costs \$15/ton, the cost of waste removal for the company is $159.5249 \cdot 15 = \$2392.87$.

41.

(a) The distance traveled is the integral of the velocity, so in T seconds you fall

$$\int_0^T 49(1 - 0.8187^t)\, dt.$$

(b) We want the number T for which

$$\int_0^T 49(1 - 0.8187^t)\, dt = 5000.$$

We can use a calculator or computer to experiment with different values for T, and we find $T \approx 107$ seconds.