Section 5.3

1. Since $f(x)$ is positive along the interval from 0 to 6 the area is simply $\int_0^6 (x^2+2)dx = 84$.

3. $\int_0^3 (4-x^2)dx \approx 7.667$.

6.

(a) The total area between $f(x)$ and the x-axis is the sum of the two given areas, so

$$\text{Area} = 7 + 6 = 13.$$

(b) To find the integral, we note that from $x = 3$ to $x = 5$, the function lies below the x-axis, and hence makes a negative contribution to the integral. So

$$\int_0^5 f(x)\,dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx = 7 - 6 = 1.$$

7. (a) Negative (b) Positive (c) Negative (d) Positive.

9.

(a) Counting the squares yields an estimate of 16.5, each with area $= 1$, so the total shaded area is approximately 16.5.

(b)

$$\int_0^8 f(x)dx = (\text{shaded area above } x\text{-axis}) - (\text{shaded area below } x\text{-axis})$$
$$\approx 6.5 - 10 = -3.5$$

(c) The answers in (a) and (b) are different because the shaded area below the x-axis is subtracted in order to find the value of the integral in (b).

13. **(a)** The area between the graph of f and the x-axis between $x = a$ and $x = b$ is 13, so

$$\int_a^b f(x)\,dx = 13.$$

(b) Since the graph of $f(x)$ is below the x-axis for $b < x < c$,

$$\int_b^c f(x)\,dx = -2.$$

(c) Since the graph of $f(x)$ is above the x-axis for $a < x < b$ and below for $b < x < c$,

$$\int_a^c f(x)\,dx = 13 - 2 = 11.$$

(d) The graph of $|f(x)|$ is the same as the graph of $f(x)$ except that the part below the x-axis is reflected to be above it. See Figure 5.31. Thus

$$\int_a^c |f(x)|\,dx = 13 + 2 = 15.$$

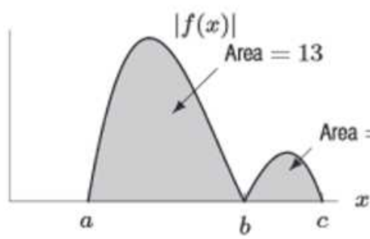

Figure 5.31

19. **(a)** $\displaystyle\int_{-3}^{0} f(x)\,dx = -2.$

(b) $\displaystyle\int_{-3}^{4} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx = -2 + 2 - \frac{A}{2} = -\frac{A}{2}.$

20. It appears that $f(x)$ is positive on the whole interval $0 \le x \le 20$, so we have

$$\text{Area} = \int_0^{20} f(x)dx.$$

We estimate the value of the integral using left and right sums:

$$\text{Left-hand sum} = 15\cdot 5 + 18\cdot 5 + 20\cdot 5 + 16\cdot 5 = 345.$$
$$\text{Right-hand sum} = 18\cdot 5 + 20\cdot 5 + 16\cdot 5 + 12\cdot 5 = 330.$$

A better estimate of the area is the average of the two:

$$\text{Area} \approx \frac{345 + 330}{2} = 337.5.$$

23. A graph of $y = 2\cos(t/10)$ shows that this function is nonnegative on the interval $t = 1$ to $t = 2$. Thus,

$$\text{Area} = \int_1^2 2\cos\frac{t}{10}\,dt = 1.977.$$

The integral was evaluated on a calculator.

27.

The graph of $y = x^4 - 8$ has intercepts $x = \pm\sqrt[4]{8}$. See Figure 5.40. Since the region is below the x-axis, the integral is negative, so

$$\text{Area} = -\int_{-\sqrt[4]{8}}^{\sqrt[4]{8}} (x^4 - 8)\,dx = 21.527.$$

The integral was evaluated on a calculator.

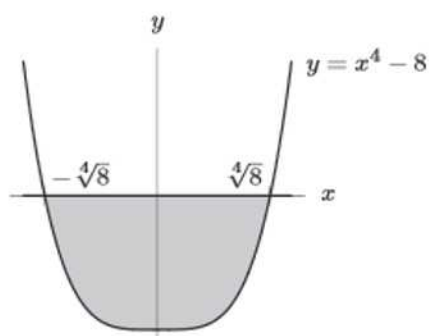

Figure 5.40