Section 5.1

1.

(a) The velocity is 30 miles/hour for the first 2 hours, 40 miles/hour for the next $1/2$ hour, and 20 miles/hour for the last 4 hours. The entire trip lasts $2 + 1/2 + 4 = 6.5$ hours, so we need a scale on our horizontal (time) axis running from 0 to 6.5. Between $t = 0$ and $t = 2$, the velocity is constant at 30 miles/hour, so the velocity graph is a horizontal line at 30. Likewise, between $t = 2$ and $t = 2.5$, the velocity graph is a horizontal line at 40, and between $t = 2.5$ and $t = 6.5$, the velocity graph is a horizontal line at 20. The graph is shown in Figure 5.1.

(b) How can we visualize distance traveled on the velocity graph given in Figure 5.1? The velocity graph looks like the top edges of three rectangles. The distance traveled on the first leg of the journey is (30 miles/hour)(2 hours), which is the height times the width of the first rectangle in the velocity graph. The distance traveled on the first leg of the trip is equal to the area of the first rectangle. Likewise, the distances traveled during the second and third legs of the trip are equal to the areas of the second and third rectangles in the velocity graph. It appears that distance traveled is equal to the area under the velocity graph.

In Figure 5.2, the area under the velocity graph in Figure 5.1 is shaded. Since this area is three rectangles and the area of each rectangle is given by Height × Width, we have

$$\begin{aligned}\text{Total area} &= (30)(2) + (40)(1/2) + (20)(4)\\ &= 60 + 20 + 80 = 160.\end{aligned}$$

The area under the velocity graph is equal to distance traveled.

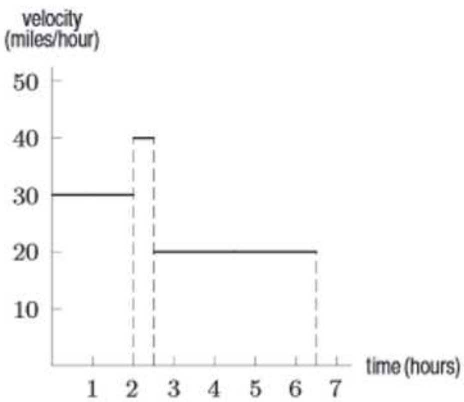

Figure 5.1: Velocity graph

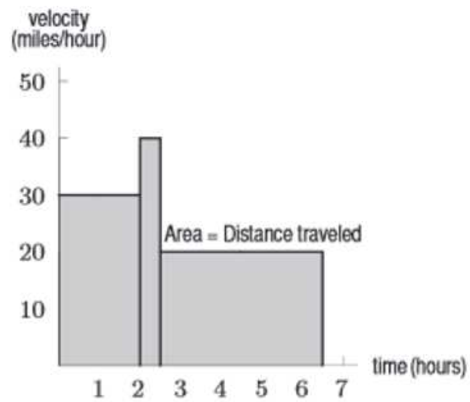

Figure 5.2: The area under the velocity graph gives distance traveled

3. (a) Right sum

(b) Since the highlighted area is smaller than the area under the curve, the sum gives an underestimate.

(c) n=5

(d) 15/5=3.

(e) The area of each rectangle is 4x3=12. There are approximately 13.5 rectangles with total area of 13.5x12 = 162.

5. We use Distance = Rate $\times$ Time on each subinterval with $\Delta t = 3$.

$$\text{Underestimate} = 0 \cdot 3 + 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 = 240,$$
$$\text{Overestimate} = 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 + 75 \cdot 3 = 465.$$

We know that

$$240 \leq \text{Distance traveled} \leq 465.$$

A better estimate is the average. We have

$$\text{Distance traveled} \approx \frac{240 + 465}{2} = 352.5.$$

The car travels about 352.5 feet during these 12 seconds.

7.

Just counting the squares (each of which has area 10), and allowing for the broken squares, we can see that the area under the curve from 0 to 6 is between 140 and 150. Hence the distance traveled is between 140 and 150 meters.

11.

Figure 5.4 shows the graph of $f(t)$. The region under the graph of $f(t)$ from $t = 0$ to $t = 10$ is a triangle of base 10 seconds and height 50 meter/sec. Then

$$\text{Distance traveled} = \text{Area of triangle} = \frac{1}{2} \cdot 10 \cdot 50 = 250 \text{ meters.}$$

Thus the distance traveled is 250 meters.

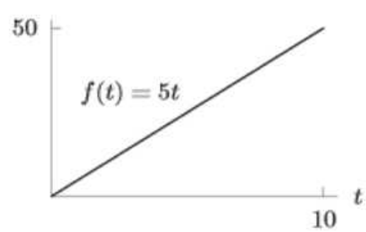

Figure 5.4

14.

(a) See Figure 5.6.

(b) The distance traveled is the area under the graph of the velocity in Figure 5.6. The region is a triangle of base 5 seconds and altitude 50 ft/sec, so the distance traveled is $(1/2)5 \cdot 50 = 125$ feet.

(c) The slope of the graph of the velocity function is the same, so the triangular region under it has twice the altitude and twice the base (it takes twice as long to stop). See Figure 5.7. Thus, the area is 4 times as large and the car travels 4 times as far.

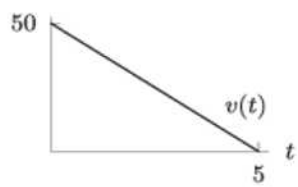

Figure 5.6

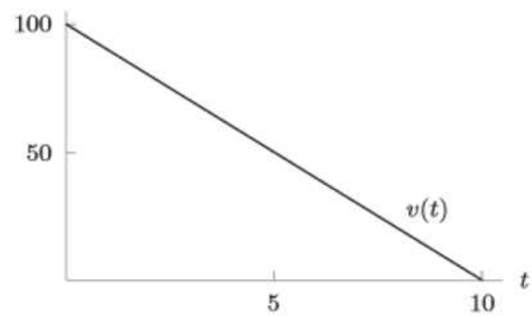

Figure 5.7

17.

(a) Car A has the largest maximum velocity because the peak of car A's velocity curve is higher than the peak of B's.

(b) Car A stops first because the curve representing its velocity hits zero (on the t-axis) first.

(c) Car B travels farther because the area under car B's velocity curve is the larger.

21.

(a) Based on the data, we will calculate the underestimate and the overestimate of the total change. A good estimate will be the average of both results.

Underestimate of total change

$$= 37 \cdot 10 + 41 \cdot 10 + 77 \cdot 10 + 77 \cdot 10 + 79 \cdot 10 = 3110.$$

77 was considered twice since we needed to calculate the area under the graph.

Overestimate of total change

$$= 41 \cdot 10 + 78 \cdot 10 + 78 \cdot 10 + 86 \cdot 10 + 86 \cdot 10 = 3690.$$

78 and 86 were considered twice since we needed to calculate the area over the graph.

The average is: $(3110 + 3690)/2 = 3400$ million people.

(b) The total change in the world's population between 1950 and 2000 is given by the difference between the populations in those two years. That is, the change in population equals

$$6085 \text{ (population in 2000)} - 2555 \text{ (population in 1950)} = 3530 \text{ million people.}$$

Our estimate of 3400 million people and the actual difference of 3530 million people are close to each other, suggesting our estimate was a good one.