Section 4.6

1.

The effect on the quantity demanded is approximately E times the change in price. A price increase causes a decrease in quantity demanded and a price decrease causes an increase in quantity demanded.

(a) The quantity demanded decreases by about $0.5(3\%) = 1.5\%$.
(b) The quantity demanded increases by about $0.5(3\%) = 1.5\%$.

3.

(a) We have $E = |p/q \cdot dq/dp| = |\text{dollars/tons} \cdot \text{tons/dollars}|$. All the units cancel, and so elasticity has no units.
(b) We have $E = |p/q \cdot dq/dp| = |\text{yen/liters} \cdot \text{liters/yen}|$. All the units cancel, and so elasticity has no units.
(c) Elasticity has no units. This is why it makes sense to compare elasticities of different products valued in different ways and measured in different units. Changing units of measurement will not change the value of elasticity.

5.

Demand for high-definition TV's will be elastic, since it is not a necessary item. If the prices are too high, people will not choose to buy them, so price changes will cause relatively large demand changes.

9.

The elasticity of demand for a product, E, is given by

$$E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|.$$

We first find $dq/dp = -4p$. At a price of \$5, the quantity demanded is $q = 200 - 50 = 150$ and $dq/dp = -20$, so

$$E = \left|\frac{5}{150} \cdot (-20)\right| = \frac{2}{3}.$$

Since $E < 1$ demand is inelastic.

12.

(a) If the price of yams is \$2/pound, the quantity sold will be

$$q = 5000 - 10(2)^2 = 5000 - 40 = 4960$$

so 4960 pounds will be sold.

(b) Elasticity of demand is given by

$$E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right| = \left|\frac{p}{q} \cdot \frac{d}{dp}(5000 - 10p^2)\right| = \left|\frac{p}{q} \cdot (-20p)\right| = \frac{20p^2}{q}$$

Substituting $p = 2$ and $q = 4960$ yields

$$E = \frac{20(2)^2}{4960} = \frac{80}{4960} = 0.016.$$

Since $E < 1$ the demand is inelastic, so it would be more accurate to say "People want yams and will buy them no matter what the price."

13.

(a) At a price of \$2/pound, the quantity sold is

$$q = 5000 - 10(2)^2 = 5000 - 40 = 4960$$

so the total revenue is

$$R = pq = 2 \cdot 4960 = \$9{,}920$$

(b) We know that $R = pq$, and that $q = 5000 - 10p^2$, so we can substitute for q to find $R(p)$

$$R(p) = p(5000 - 10p^2) = 5000p - 10p^3$$

To find the price that maximizes revenue we take the derivative and set it equal to 0.

$$\begin{aligned} R'(p) &= 0 \\ 5000 - 30p^2 &= 0 \\ 30p^2 &= 5000 \\ p^2 &= 166.67 \\ p &= \pm 12.91 \end{aligned}$$

We disregard the negative answer, so $p = 12.91$ is the only critical point. Is it the maximum? We use the first derivative test.

$$\begin{aligned} &R'(p) > 0 \ \text{ if } \ p < 12.91 \text{ and} \\ &R'(p) < 0 \ \text{ if } \ p > 12.91 \end{aligned}$$

So $R(p)$ has a local maximum at $p = 12.91$. We also test the function at $p = 0$, which is the only endpoint.

$$R(0) = 5000(0) - 10(0)^3 = 0$$

$$R(12.91) = 5000(12.91) - 10(12.91)^3 = 64{,}550 - 21{,}516.85 = \$43{,}033.15$$

So we conclude that revenue is maximized at price of \$12.91/pound.

(c) At a price of \$12.91/pound the quantity sold is

$$q = 5000 - 10(12.91)^2 = 5000 - 1666.68 = 3333.32$$

so the total revenue is

$$R = pq = (3333.32)(12.91) = \$43{,}033.16$$

which agrees with part (b).

(d)

$$E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right| = \left|\frac{p}{q} \cdot \frac{d}{dp}(5000 - 10p^2)\right| = \left|\frac{p}{q} \cdot (-20p)\right| = \frac{20p^2}{q}$$

Substituting $p = 12.91$ and $q = 3333.32$ yields

$$E = \frac{20(12.91)^2}{3333.32} = \frac{3333.36}{3333.32} \approx 1$$

which agrees with the result that maximum revenue occurs when $E = 1$.

15. The revenue is maximized by finding the critical point of the revenue function:

$$R = pq = p(1000 - 2p^2) = 1000p - 2p^3.$$

Differentiate to find the critical points:

$$\frac{dR}{dp} = 1000 - 6p^2 = 0$$
$$p^2 = \frac{1000}{6}$$
$$p \approx 12.91$$

To maximize revenues, the price of the product should be \$12.91.

16.

Demand is elastic at all prices. No matter what the price is, you can increase revenue by lowering the price. In the end, you would lower your prices all the way to zero. This is not a realistic example, but it is mathematically possible. It would correspond, for instance, to the demand equation $q = 1/p^2$, which gives revenue $R = pq = 1/p$ which is decreasing for all prices $p > 0$.

17.

Demand is inelastic at all prices. No matter what the price is, you can increase revenue by raising the price, so there is no actual price for which your revenue is maximized. This is not a realistic example, but it is mathematically possible. It would correspond, for instance, to the demand equation $q = 1/\sqrt{p}$, which gives revenue $R = pq = \sqrt{p}$ which is increasing for all prices $p > 0$.