Section4.5

1.

(a) Since the graph is concave down, the average cost gets smaller as q increases. This is because the cost per item gets smaller as q increases. There is no value of q for which the average cost is minimized since for any q_0 larger than q the average cost at q_0 is less than the average cost at q. Graphically, the average cost at q is the slope of the line going through the origin and through the point $(q, C(q))$. Figure 4.104 shows how as q gets larger, the average cost decreases.

(b) The average cost will be minimized at some q for which the line through $(0, 0)$ and $(q, c(q))$ is tangent to the cost curve. This point is shown in Figure 4.105.

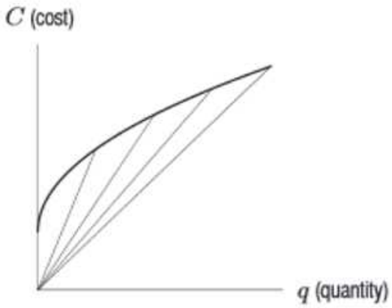

Figure 4.104

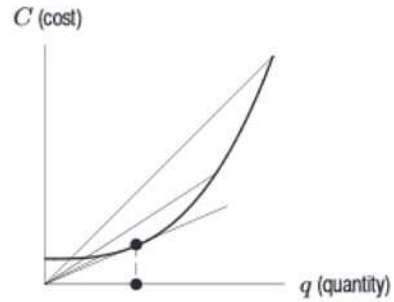

Figure 4.105

3.

(a) (i) The average cost of quantity q is given by the formula $C(q)/q$. So average cost at $q = 25$ is given by $C(25)/25$. From the graph, we see that $C(25) \approx 200$, so $a(q) \approx 200/25 \approx \$8$ per unit. To interpret this graphically, note that $a(q) = C(q)/q = (C(q) - 0)/(q - 0)$. This is the formula for the slope of a line from the origin to a point $(q, C(q))$ on the curve. So $a(25)$ is the slope of a line connecting $(0, 0)$ to $(25, C(25))$. See Figure 4.106.

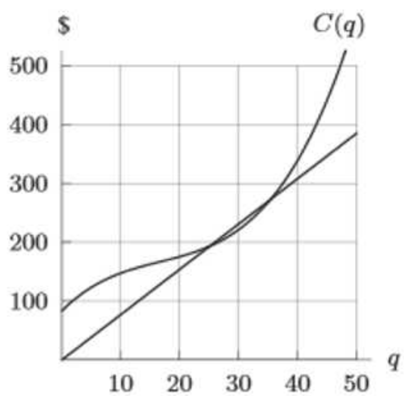

Figure 4.106

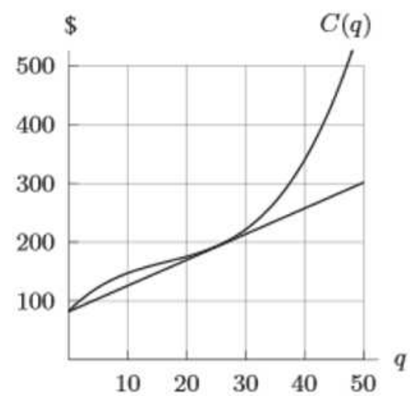

Figure 4.107

(ii) The marginal cost is $C'(q)$. This derivative is the slope of the tangent line to $C(q)$ at $q = 25$. To estimate this slope, we draw the tangent line, shown in Figure 4.107. From this plot, we see that the points $(50, 300)$ and $(0, 100)$ are approximately on this line, so its slope is approximately $(300 - 100)/(50 - 0) = 4$. Thus, $C'(25) \approx \$4$ per unit.

(b) We know that $a(q)$ is minimized where $a(q) = C'(q)$. Using the graphical interpretations from parts (i) and (ii), we see that $a(q)$ is minimized where the line passing from $(q, C(q))$ to the origin is also tangent to the curve. To find such points, a variety of lines passing through the origin and the curve are shown in Figure 4.108. The line which is also a tangent touches the curve at $q \approx 30$. So $q \approx 30$ units minimizes $a(q)$.

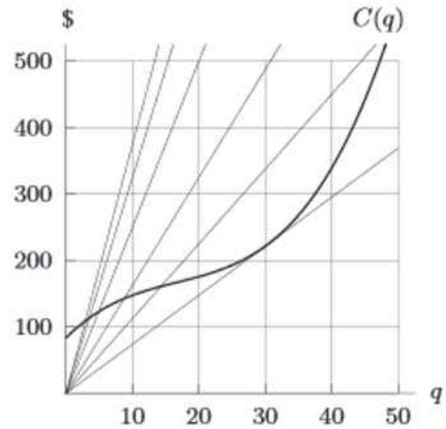

Figure 4.108

5.

The cost function is $C(q) = 1000 + 20q$. The marginal cost function is the derivative $C'(q) = 20$, so the marginal cost to produce the 200^{th} unit is \$20 per unit. The average cost of producing 200 units is given by

$$a(200) = \frac{C(200)}{200} = \frac{5000}{200} = \$25/\text{unit}$$

7.

(a) The line connecting the origin and the graph of $C(q)$ in Figure 4.112 appears to have minimum slope at $q = 6$. Therefore we conclude that average cost is minimized at about $q = 6$.

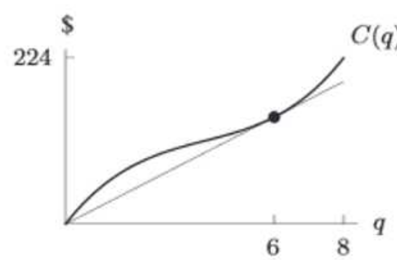

Figure 4.112

(b) The average cost of the first q items is given by

$$a(q) = \frac{C(q)}{q} = \frac{q^3 - 12q^2 + 60q}{q} = q^2 - 12q + 60$$

We want to minimize $a(q)$. Differentiating gives

$$a'(q) = 2q - 12.$$

Setting this equal to 0 and solving yields $q = 6$. Is this our minimum? We have $a'(q) < 0$ if $q < 6$ and $a'(q) > 0$ if $q > 6$, so $q = 6$ is a local minimum for $a(q)$. From Figure 4.112, we see that $q = 6$ is the global minimum for $0 \leq q \leq 8$.

9.

(a) $a(q) = C(q)/q$, so $C(q) = 0.01q^3 - 0.6q^2 + 13q$.

(b) Taking the derivative of $C(q)$ gives an expression for the marginal cost:

$$C'(q) = MC(q) = 0.03q^2 - 1.2q + 13.$$

To find the smallest MC we take its derivative and find the value of q that makes it zero. So: $MC'(q) = 0.06q - 1.2 = 0$ when $q = 1.2/0.06 = 20$. This value of q must give a minimum because the graph of $MC(q)$ is a parabola opening upward. Therefore the minimum marginal cost is $MC(20) = 1$. So the marginal cost is at a minimum when the additional cost per item is \$1.

(c) $a'(q) = 0.02q - 0.6$

Setting $a'(q) = 0$ and solving for q gives $q = 30$ as the quantity at which the average is minimized, since the graph of a is a parabola which opens upward. The minimum average cost is $a(30) = 4$ dollars per item.

(d) The marginal cost at $q = 30$ is $MC(30) = 0.03(30)^2 - 1.2(30) + 13 = 4$. This is the same as the average cost at this quantity. Note that since $a(q) = C(q)/q$, we have $a'(q) = (qC'(q) - C(q))/q^2$. At a critical point, q_0, of $a(q)$, we have

$$0 = a'(q_0) = \frac{q_0 C'(q_0) - C(q_0)}{q_0^2},$$

so $C'(q_0) = C(q_0)/q_0 = a(q_0)$. Therefore $C'(30) = a(30) = 4$ dollars per item.

Another way to see why the marginal cost at $q = 30$ must equal the minimum average cost $a(30) = 4$ is to view $C'(30)$ as the approximate cost of producing the 30^{th} or 31^{st} good. If $C'(30) < a(30)$, then producing the 31^{st} good would lower the average cost, i.e. $a(31) < a(30)$. If $C'(30) > a(30)$, then producing the 30^{th} good would raise the average cost, i.e. $a(30) > a(29)$. Since $a(30)$ is the global minimum, we must have $C'(30) = a(30)$.

10.

(a) The marginal cost tells us that additional units produced would cost about \$10 each, which is below the average cost, so producing them would reduce average cost.

(b) It is impossible to determine the effect on profit from the information given. Profit depends on both cost and revenue, $\pi = R - C$, but we have no information on revenue.