Section 4.4

1.

The profit function is positive when $R(q) > C(q)$, and negative when $C(q) > R(q)$. It's positive for $5.5 < q < 12.5$, and negative for $0 < q < 5.5$ and $12.5 < q$. Profit is maximized when $R(q) > C(q)$ and $R'(q) = C'(q)$ which occurs at about $q = 9.5$. See Figure 4.95.

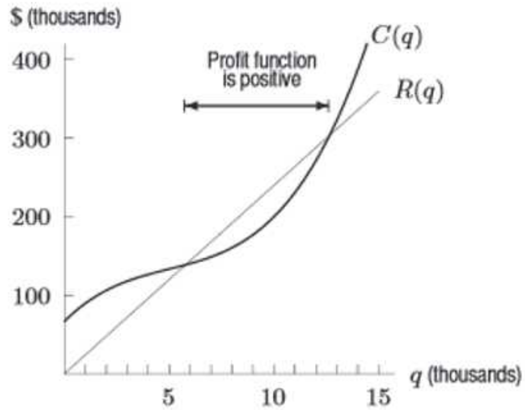

Figure 4.95

3.

First find marginal revenue and marginal cost.

$$MR = R'(q) = 450$$

$$MC = C'(q) = 6q$$

Setting $MR = MC$ yields $6q = 450$, so marginal cost is equal to marginal revenue v

$$q = \frac{450}{6} = 75 \text{ units.}$$

Is profit maximized at $q = 75$? Profit $= R(q) - C(q)$;

$$\begin{aligned} R(75) - C(75) &= 450(75) - (10{,}000 + 3(75)^2) \\ &= 33{,}750 - 26{,}875 = \$6875. \end{aligned}$$

Testing $q = 74$ and $q = 76$:

$$\begin{aligned} R(74) - C(74) &= 450(74) - (10{,}000 + 3(74)^2) \\ &= 33{,}300 - 26{,}428 = \$6872. \end{aligned}$$

$$\begin{aligned} R(76) - C(76) &= 450(76) - (10{,}000 + 3(76)^2) \\ &= 34{,}200 - 27{,}328 = \$6872. \end{aligned}$$

Since profit at $q = 75$ is more than profit at $q = 74$ and $q = 76$, we conclude that profit is maximized locally at $q = 75$. The only endpoint we need to check is $q = 0$.

$$\begin{aligned} R(0) - C(0) &= 450(0) - (10{,}000 + 3(0)^2) \\ &= -\$10{,}000. \end{aligned}$$

This is clearly not a maximum, so we conclude that the profit is maximized globally at $q = 75$, and the total profit at this production level is \$6,875.

7.

(a) The profit earned by the 51^{st} is the revenue earned by the 51^{st} item minus the cost of producing the 51^{st} item. This can be approximated by

$$\pi'(50) = R'(50) - C'(50) = 84 - 75 = \$9.$$

Thus the profit earned from the 51^{st} item will be approximately \$9.

(b) The profit earned by the 91^{st} item will be the revenue earned by the 91^{st} item minus the cost of producing the 91^{st} item. This can be approximated by

$$\pi'(90) = R'(90) - C'(90) = 68 - 71 = -\$3.$$

Thus, approximately three dollars are lost in the production of the 91^{st} item.

(c) If $R'(78) > C'(78)$, production of a 79^{th} item would increase profit. If $R'(78) < C'(78)$, production of one less item would increase profit. Since profit is maximized at $q = 78$, we must have

$$C'(78) = R'(78).$$

9.

(a) At $q = 5000$, $MR > MC$, so the marginal revenue to produce the next item is greater than the marginal cost. This means that the company will make money by producing additional units, and production should be increased.

(b) Profit is maximized where $MR = MC$, and where the profit function is going from increasing ($MR > MC$) to decreasing ($MR < MC$). This occurs at $q = 8000$.

11.

The company should increase production if $MR > MC$, since increasing production then adds more to revenue than to cost—a net gain for the company.

(a) Since $MC(25) = 17.75$ and $MR(25) = 30$, the company should increase production.

(b) Since $MC(50) = 39$ and $MR(50) = 30$, the company should decrease production.

(c) Since $MC(80) = 114$ and $MR(80) = 30$, the company should decrease production.

13.

The profit is maximized at the point where the difference between revenue and cost is greatest. Thus the profit is maximized at approximately $q = 4000$.

19.

(a) If $q = 3000$, the demand equation gives $p = 70 - 0.02 \cdot 3000 = 10$. That is, at a price of \$10, 3000 people attend. At this price,

$$\text{Revenue} = 3000 \text{ people} \cdot 10 \text{ dollars/person} = \$30{,}000.$$

To find total revenue at a price of \$20, first find the attendance at this price. Substituting $p = 20$ into the demand equation, $p = 70 - 0.02q$, gives

$$20 = 70 - 0.02q.$$

Solving for q, we get

$$-50 = -0.02q$$

$$2500 = q.$$

That is, at a price of \$20, attendance is 2500 people, and

$$\text{Revenue} = 2500 \cdot 20 = \$50,000.$$

Notice that, although demand is reduced, the revenue is higher at a price of \$20 than at \$10.

(b) Since Revenue = Price × Quantity $= p \cdot q$ and $p = 70 - 0.002q$, we have

$$\begin{aligned} R(q) &= (70 - 0.02q)q \\ &= 70q - 0.02q^2. \end{aligned}$$

(c) To maximize revenue, find the critical points of the revenue function $R(q) = 70q - 0.02q^2$:

$$\begin{aligned} R'(q) &= 70 - 0.02 \cdot 2q \\ 0 &= 70 - 0.04q \\ 70 &= 0.04q \\ 1750 &= q. \end{aligned}$$

The graph of revenue is a parabola opening downward, so an attendance of 1750 gives the maximum revenue.

(d) Using the demand equation, we find the price corresponding to an attendance of 1750:

$$p = 70 - 0.02 \cdot 1750 = 70 - 35 = 35.$$

The optimal price for a ticket at the amusement park is \$35.

(e) When the optimal price of \$35 is charged, the attendance at the park is 1750 people. Thus, the maximum revenue is $R = pq = 35 \cdot 1750 = \$61{,}250$. The corresponding profit cannot be determined without knowing the costs.

21.

We first need to find an expression for revenue in terms of price. At a price of \$8, 1500 tickets are sold. For each \$1 above \$8, 75 fewer tickets are sold. This suggests the following formula for q, the quantity sold for any price p.

$$\begin{aligned} q &= 1500 - 75(p - 8) \\ &= 1500 - 75p + 600 \\ &= 2100 - 75p. \end{aligned}$$

We know that $R = pq$, so substitution yields

$$R(p) = p(2100 - 75p) = 2100p - 75p^2$$

To maximize revenue, we find the derivative of $R(p)$ and set it equal to 0.

$$\begin{aligned} R'(p) &= 2100 - 150p = 0 \\ 150p &= 2100 \end{aligned}$$

so $p = \frac{2100}{150} = 14$. Does $R(p)$ have a maximum at $p = 14$? Using the first derivative test,

$$\begin{aligned} R'(p) &> 0 \text{ if } p < 14 \text{ and} \\ R'(p) &< 0 \text{ if } p > 14. \end{aligned}$$

So $R(p)$ has a local maximum at $p = 14$. Since this is the only critical point for $p \geq 0$, it must be a global maximum. So we conclude that revenue is maximized when the price is \$14.