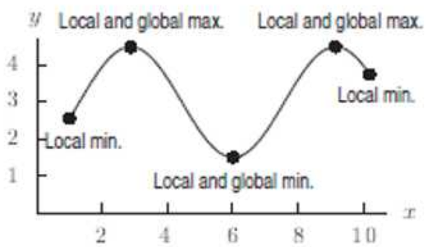
Section 4.3

1.

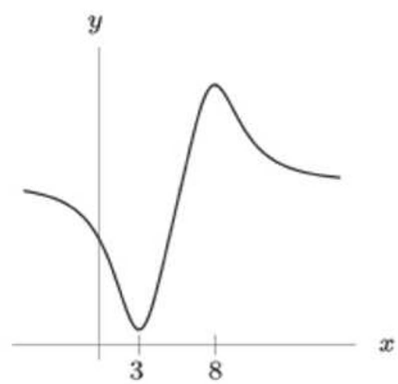
2.



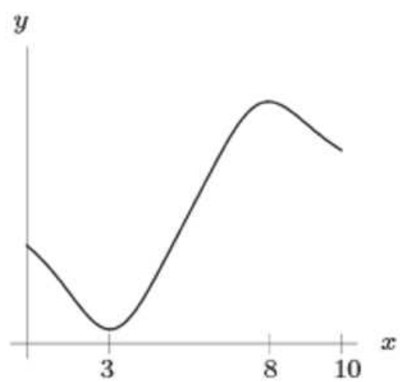
The global maximum is achieved at the two local maxima, which are at the same height.

3. **(a)** (IV)
(b) (I)
(c) (III)
(d) (II)

7.



13.



18.

(a) Differentiating $f(x) = x^3 - 3x^2 - 9x + 15$ produces $f'(x) = 3x^2 - 6x - 9$. A second differentiation produces $f''(x) = 6x - 6$.
(b) $f'(x)$ is defined for all x and $f'(x) = 0$ when $x = -1, 3$. Thus $x = -1, 3$ are critical points.
(c) $f''(x)$ is defined for all x and $f''(x) = 0$ when $x = 1$. Since the concavity of f changes at this point, it is an inflection point.

(d) $f(-5) = -140$, $f(4) = -5$, $f(-1) = 2$, $f(3) = -12$. So f has a global maximum at $x = -1$ and a global minimum at $x = -5$, and a local minimum at $x = 3$
(e) Plotting the function $f(x)$ for $-5 \le x \le 4$ gives the graph shown in Figure 4.76:

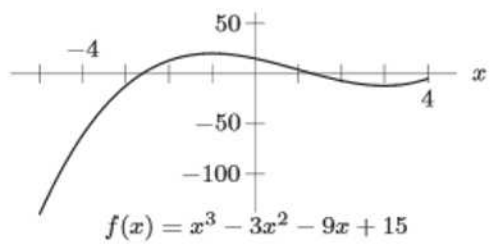


Figure 4.76

27.

This is a parabola opening downward. We find the critical points by setting $g'(x) = 0$:

$$\begin{aligned} g'(x) = 4 - 2x &= 0 \\ x &= 2. \end{aligned}$$

Since $g'(x) > 0$ for $x < 2$ and $g'(x) < 0$ for $x > 2$, the critical point at $x = 2$ is a local maximum.

As $x \to \pm\infty$, the value of $g(x) \to -\infty$. Thus, the local maximum at $x = 2$ is a global maximum of $g(2) = 4 \cdot 2 - 2^2 - 5 = -1$. There is no global minimum. See Figure 4.79.

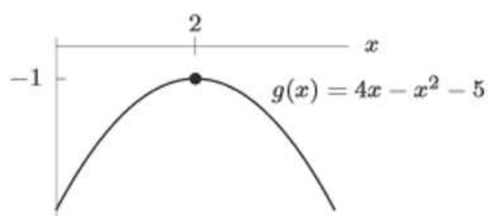


Figure 4.79

31.

Differentiating using the quotient rule gives

$$f'(t) = \frac{1(1+t^2) - t(2t)}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2}.$$

The critical points are the solutions to

$$\begin{aligned} \frac{1-t^2}{(1+t^2)^2} &= 0 \\ t^2 &= 1 \\ t &= \pm 1. \end{aligned}$$

Since $f'(t) > 0$ for $-1 < t < 1$ and $f'(t) < 0$ otherwise, there is a local minimum at $t = -1$ and a local maximum at $t = 1$.

As $t \to \pm\infty$, we have $f(t) \to 0$. Thus, the local maximum at $t = 1$ is a global maximum of $f(1) = 1/2$, and the local minimum at $t = -1$ is a global minimum of $f(-1) = -1/2$. See Figure 4.83.

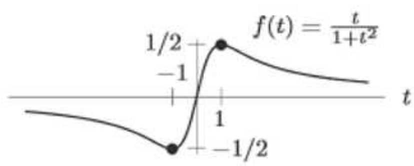


Figure 4.83

39.

Rewriting the expression for I using the properties of logs gives

$$I = 192(\ln S - \ln 762) - S + 763.$$

Differentiating with respect to S gives

$$\frac{dI}{dS} = \frac{192}{S} - 1.$$

At a critical point

$$\begin{aligned} \frac{192}{S} - 1 &= 0 \\ S &= 192. \end{aligned}$$

Since

$$\frac{d^2I}{dS^2} = -\frac{192}{S^2},$$

we see that if $S = 192$, we have $d^2I/dS^2 < 0$, so $S = 192$ is a local maximum. From Figure 4.89, we see that it is a global maximum. The maximum possible number of infected children is therefore

$$I = 192 \ln\left(\frac{192}{762}\right) - 192 + 763 = 306 \text{ children.}$$

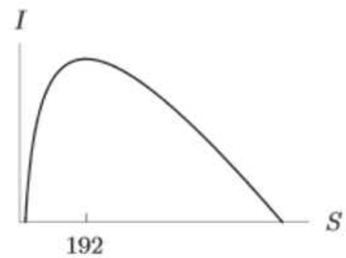


Figure 4.89