Section 4.2

1. We find an inflection point by noting where the concavity changes. Such points are shown in Figure 4.29. There are two inflection points.

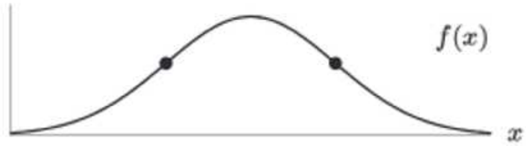


Figure 4.29

2. We find an inflection point by noting where the concavity changes. Such points are shown in Figure 4.31: There are two inflection points.

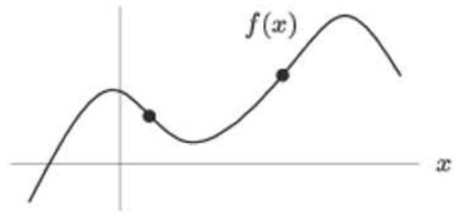


Figure 4.31

3. We find an inflection point by noting where the concavity changes. Such points are shown in Figure 4.28. There are three inflection points.

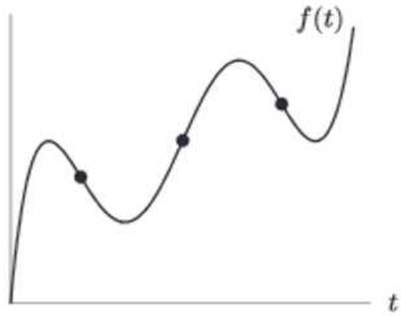


Figure 4.28

4. We find an inflection point by noting where the concavity changes. Looking at Figure 4.30, we see that in fact the concavity changes only at the critical point. So there is one inflection point, which is also a critical point.

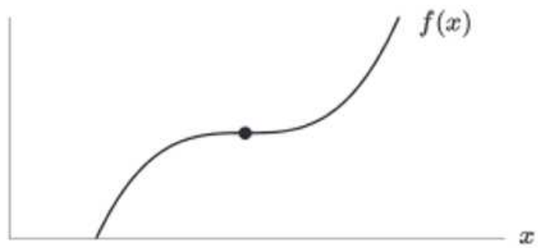
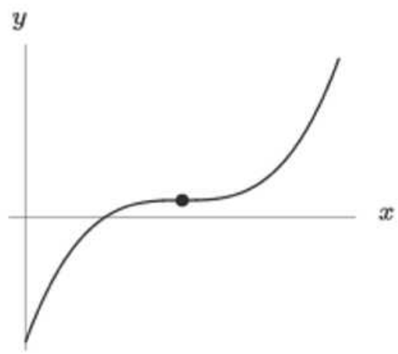


Figure 4.30

7.

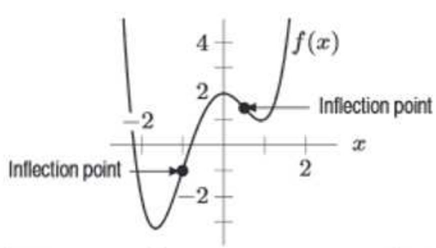


11.

From the graph of $f(x)$ in the figure below, we see that the function must have two inflection points. We calculate $f'(x) = 4x^3 + 3x^2 - 6x$, and $f''(x) = 12x^2 + 6x - 6$. Solving $f''(x) = 0$ we find that:

$$x_1 = -1 \quad \text{and} \quad x_2 = \frac{1}{2}.$$

Since $f''(x) > 0$ for $x < x_1$, $f''(x) < 0$ for $x_1 < x < x_2$, and $f''(x) > 0$ for $x_2 < x$, it follows that both points are inflection points.



13.

A critical point will occur whenever $f'(x) = 0$ or f' is undefined. Since $f'(x)$ is always defined, we set

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0.$$

Factoring, we get

$$f'(x) = 3(x - 1)(x + 1) = 0.$$

So, $x = 1$ or $x = -1$. To find the inflection points of $f(x)$, we find where $f''(x)$ goes from negative to positive or vice versa. For a point to satisfy this condition, it must have at least $f''(x) = 0$ or f'' undefined. Since $f''(x) = 6x$, we know $f''(x)$ is always defined. It is zero when $6x = 0$, so $x = 0$. Since $f''(x) = 6x$ is negative for $x < 0$ and positive for $x > 0$, $x = 0$ must be an inflection point for $f(x)$.

So $x = 1$ and $x = -1$ are critical points of $f(x)$, and $x = 0$ is an inflection point for $f(x)$.

To identify the nature of the critical points $x = 1$ and $x = -1$ that we have found, we can look at a graph of $f(x)$ for values of x near the critical points. Such a graph is shown in Figure 4.39. From the graph we see that $f(-1)$ is a local maximum of f and $f(1)$ is a local minimum of f.

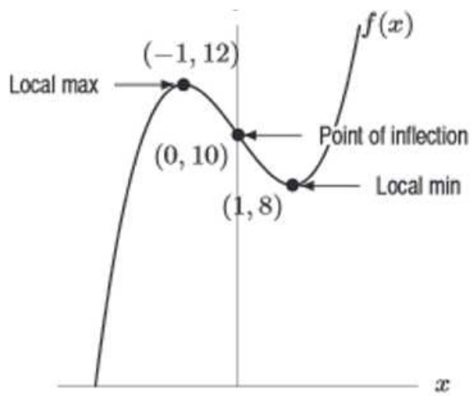


Figure 4.39

19.

We first find the critical points of $f(x) = x^4 - 4x^3 + 10$. Since $f'(x) = 4x^3 - 12x^2$, setting the derivative equal to 0 and factoring yields

$$4x^3 - 12x^2 = 0$$
$$4x^2(x-3) = 0$$

So $x = 0$ and $x = 3$ are the critical points of $f(x)$.

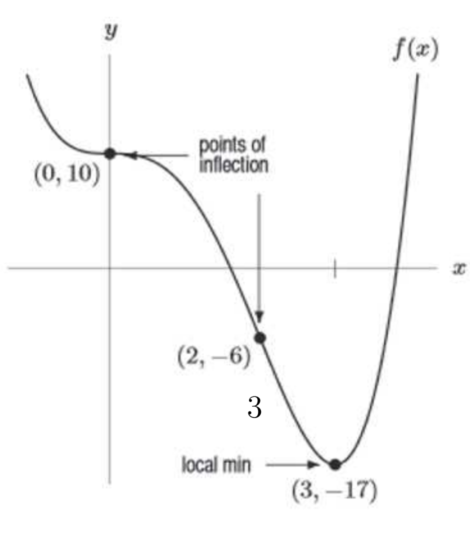
We now find the inflection points of $f(x)$. Since

$$f''(x) = 12x^2 - 24x,$$

setting the second derivative equal to 0 and factoring yields

$$12x^2 - 24x = 0$$
$$12x(x-2) = 0.$$

So $x = 0$ and $x = 2$ may be points of inflection of $f(x)$. Since $f''(x)$ changes sign at both $x = 0$ and $x = 2$, both are points of inflection for $f(x)$. From Figure 4.45, we see that the critical point $x = 3$ is a local minimum and the critical point $x = 0$ is neither a local minimum nor a local maximum.



25.

The inflection points of f are the points where f'' changes sign. If the graph shown is that of $f''(x)$, then we are looking for where the given graph passes from above the x-axis to below, or vice versa. Such points are shown in Figure 4.60:

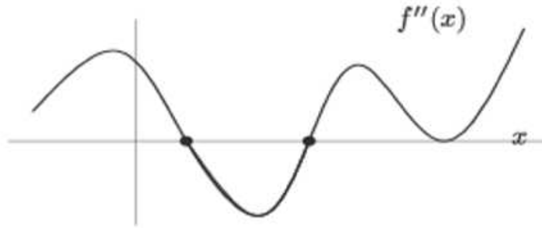
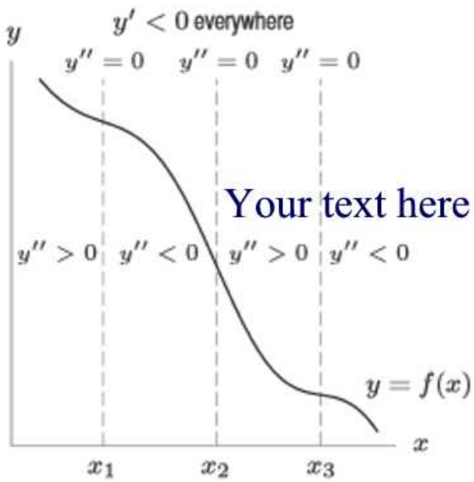


Figure 4.60

27.



29.