# Section 4.1

## 1.

We find a critical point by noting where $f'(x) = 0$ or $f'$ is undefined. Since the curve is smooth throughout, $f'$ is always defined, so we look for where $f'(x) = 0$, or equivalently where the tangent line to the graph is horizontal. These points are shown in Figure 4.1.

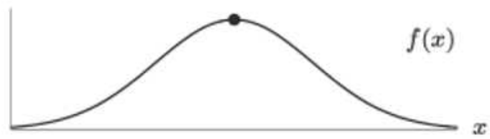



Figure 4.1

As we can see, there is one critical point. Since it is higher than nearby points, it is a local maximum.

## 2.

We find a critical point by noting where $f'(x) = 0$ or $f'$ is undefined. Since the curve is smooth throughout, $f'$ is always defined, so we look for where $f'(x) = 0$, or equivalently where the tangent line to the graph is horizontal. These points are shown in Figure 4.3:

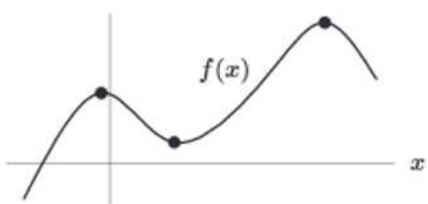



Figure 4.3

As we can see, there are three critical points. The leftmost one is a local maximum, because points near it are all lower; similarly, the middle critical point is surrounded by higher points, and is a local minimum. The critical point to the right is a local maximum.

3.

We find a critical point by noting where $f'(t) = 0$ or $f'$ is undefined. Since the curve is smooth throughout, $f'$ is always defined, so we look for where $f'(t) = 0$, or equivalently where the tangent line to the graph is horizontal. These points are shown in Figure 4.2.

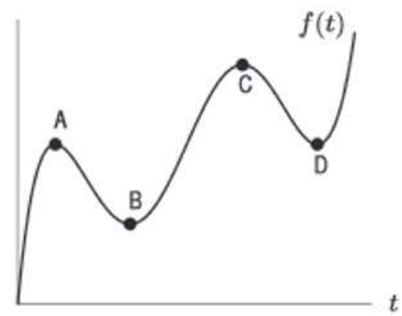



Figure 4.2

As we can see, there are four labeled critical points. Critical point A is a local maximum because points near it are all lower; similarly, point B is a local minimum, point C is a local maximum, and point D is a local minimum.

4.

We find a critical point by noting where $f'(x) = 0$ or $f'$ is undefined. Since the curve is smooth throughout, $f'$ is always defined, so we look for where $f'(x) = 0$, or equivalently where the tangent line to the graph is horizontal. These points are shown in Figure 4.4.

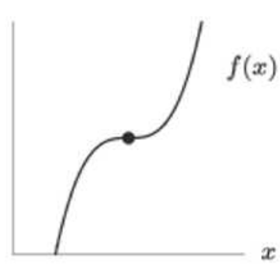


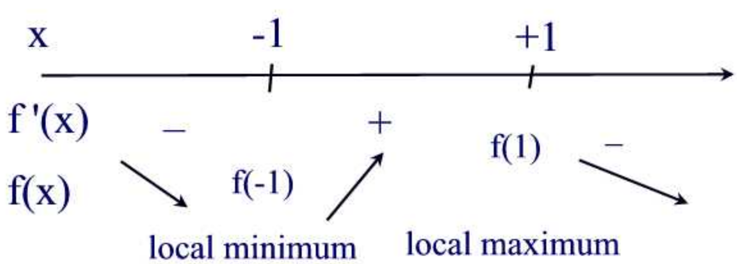

Figure 4.4

As we can see, there is one critical point. Since some nearby points (those to the left) are lower, this point is not a local minimum; since nearby points to the right are higher, it is not a local maximum. So the one critical point is neither a local minimum nor a local maximum.

15. We have $f'(x) = 1 - \frac{1}{x^2} \Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$ (critical numbers)
By the second derivative test, we find

$$f''(x) = \frac{2}{x^3} \Rightarrow f''(-1) = -2 < 0 \Rightarrow x = -1 \quad \text{(local maximum)}$$
$$f''(x) = \frac{2}{x^3} \Rightarrow f''(1) = 2 > 0 \Rightarrow x = 1 \quad \text{(local minimum)}$$

19. $f'(x) = \frac{1-x^2}{(x^2+1)^2} \Rightarrow f'(x) = 0 \Rightarrow x = \pm 1$ (critical numbers)




24. (a) f is increasing for x>0 (f '(x)>0) and decreasing for x<0 (f '(x)<0).
(b) f' (0) = 0 and f changes sign from negative to positive around 0. Thus, x=0 is a local minimum.

25. (a) f is always increasing (f '(x)>0).
(b) f '(0)=0 but f ' does not change signs around 0 so x=0 is not a local extremum.

27. (a) f is increasing for -1<x<0 and x>1. f is decreasing for x<-1 and 0<x<1.
(b) f'(-1)=0 and f' changes sign from negative to positive so that x=-1 is a local minimum. Likewise, f '(0)=0 and f ' changes sign from positive to negative so that x=0 is a local maximum. Finally, f '(1) = 0 and f ' changes sign from negative to positive so that x=1 is a local minimum.

29.

**(a)** The demand for the product is increasing when $f'(t)$ is positive, and decreasing when $f'(t)$ is negative. Inspection of the table suggests that demand is increasing during weeks 0 to 2 and weeks 6 to 10, and decreasing during weeks 3 to 5.

**(b)** Since $f'(t) = 4 > 0$ during week 2 and $f'(t) = -2 < 0$ during week 3, the demand for the product changes from increasing to decreasing near the end of week 2 or the beginning of week 3. Thus the demand has a local maximum during this time period. Since $f'(t) = -1 < 0$ during week 5 and $f'(t) = 3 > 0$ during week 6, the demand for the product changes from decreasing to increasing near the end of week 5 or the beginning of week 6. Thus the demand has a local minimum during this time period.

34.. We wish to have $f'(3) = 0$. Differentiating to find $f'(x)$ and then solving $f'(3) = 0$ for $a$ gives:

$$\begin{aligned} f'(x) &= x(ae^{ax}) + 1(e^{ax}) = e^{ax}(ax+1) \\ f'(3) &= e^{3a}(3a+1) = 0 \\ 3a+1 &= 0 \\ a &= -\frac{1}{3}. \end{aligned}$$

Thus, $f(x) = xe^{-x/3}$.