Section 3.4

7. Differentiating with respect to x, we have

$$\begin{aligned}\frac{dy}{dx} = \frac{d}{dx}(x\ln x) &= \left(\frac{d}{dx}(x)\right)\ln x + x\frac{d}{dx}(\ln x)\\ &= 1\cdot \ln x + x\cdot\frac{1}{x}\\ &= \ln x + 1.\end{aligned}$$

9. $y' = (3t^2 - 14t)e^t + (t^3 - 7t^2 + 1)e^t = (t^3 - 4t^2 - 14t + 1)e^t.$

15. $y' = 1\cdot e^{-t^2} + te^{-t^2}(-2t)$

21. Using the product and chain rules, we have

$$\begin{aligned}\frac{dz}{dt} &= 9(te^{3t} + e^{5t})^8 \cdot \frac{d}{dt}(te^{3t} + e^{5t}) = 9(te^{3t} + e^{5t})^8(1\cdot e^{3t} + t\cdot e^{3t}\cdot 3 + e^{5t}\cdot 5)\\ &= 9(te^{3t} + e^{5t})^8(e^{3t} + 3te^{3t} + 5e^{5t}).\end{aligned}$$

28. Using the quotient rule, we have

$$\frac{dy}{dz} = \frac{d}{dz}\left(\frac{1+z}{\ln z}\right) = \frac{1\cdot\ln z - (1+z)(1/z)}{(\ln z)^2} = \frac{z\ln z - 1 - z}{z(\ln z)^2}.$$

35.

$f(x) = x^2e^{-x}$, $f(0) = 0$

$f'(x) = 2xe^{-x} + x^2e^{-x}\cdot(-1) = e^{-x}(2x - x^2)$, so $f'(0) = 0$. Thus the tangent line is $y = 0$ (the x-axis). See Figure 3.11.

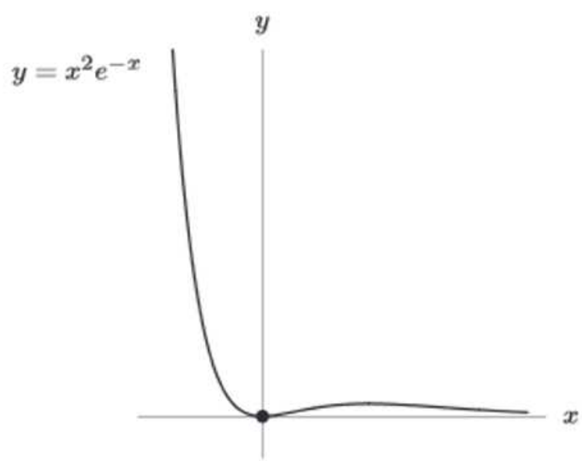

Figure 3.11

37.

(a) See Figure 3.12.

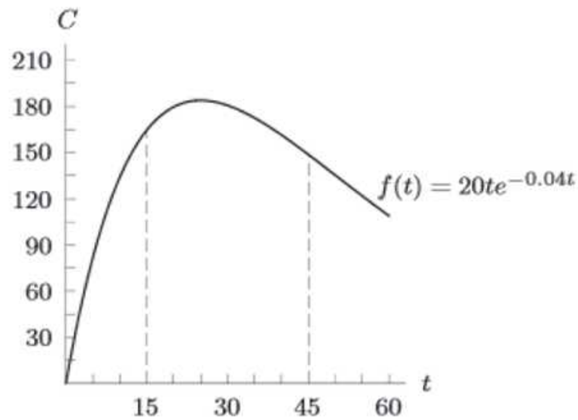

Figure 3.12

Looking at the graph of C, we can see that the see that at $t = 15$, C is increasing. Thus, the slope of the curve at that point is positive, and so $f'(15)$ is also positive. At $t = 45$, the function is decreasing, i.e. the slope of the curve is negative, and thus $f'(45)$ is negative.

(b) We begin by differentiating the function:

$$f'(t) = (20t)(-0.04e^{-0.04t}) + (e^{-0.04t})(20)$$
$$f'(t) = e^{-0.04t}(20 - 0.8t).$$

At $t = 30$,

$$f(30) = 20(30)e^{-0.04\cdot(30)} = 600e^{-1.2} \approx 181 \text{ mg/ml}$$
$$f'(30) = e^{-1.2}(20 - (0.8)(30)) = e^{-1.2}(-4) \approx -1.2 \text{ mg/ml/min}.$$

These results mean the following: At $t = 30$, or after 30 minutes, the concentration of the drug in the body ($f(30)$) is about 181 mg/ml. The rate of change of the concentration ($f'(30)$) is about -1.2 mg/ml/min, meaning that the concentration of the drug in the body is dropping by 1.2 mg/ml each minute at $t = 30$ minutes.

40.

(a) $R(p) = p \cdot 1000e^{-0.02p} = 1000pe^{-0.02p}$.

(b) $R'(p) = 1000e^{-0.02p} + 1000pe^{-0.02p}(-0.02) = e^{-0.02p}(1000 - 20p)$

(c) $R(10) = 10,000e^{-0.2} \approx 8187$; you will have about 8187 dollars in revenue if you sell the product for \$10.

$R'(10) = e^{-0.2}(1000 - 200) \approx 655$; a one dollar increase in price over \$10 will generate about \$655 in additional revenue.

41. By the product rule, $\frac{a}{dt} tf(t) = f(t) + tf'(t)$. Thus, using the information given in the problem, we have

$$f(t) + tf'(t) = 1 + f(t).$$

Subtracting $f(t)$ from both sides gives $tf'(t) = 1$, so $f'(t) = 1/t$.

42.

(a) $f(140) = 15{,}000$ says that 15,000 skateboards are sold when the cost is \$140 per board.
$f'(140) = -100$ means that if the price is increased from \$140, roughly speaking, every dollar of increase will decrease the total sales by 100 boards.

(b) $\frac{dR}{dp} = \frac{d}{dp}(p \cdot q) = \frac{d}{dp}(p \cdot f(p)) = f(p) + pf'(p)$.
So,

$$\left.\frac{dR}{dp}\right|_{p=140} = f(140) + 140f'(140)$$
$$= 15{,}000 + 140(-100) = 1000.$$

(c) From (b) we see that $\left.\frac{dR}{dp}\right|_{p=140} = 1000 > 0$. This means that the revenue will increase by about \$1000 if the price is raised by \$1.