Section 3.3

19. $y' = 1/(x\ln(x))$ 22. $\frac{dy}{dx} = 2(5+e^x)e^x$.

29.

We have $f(2) = \ln(2^2+1) = \ln(5) = 1.609$. We have $f'(t) = (2t)/(t^2+1)$, so $f'(2) = 4/5 = 0.8$. The relative rate of change at $t = 2$ is

$$\frac{f'(2)}{f(2)} = \frac{0.8}{1.609} = 0.497 = 49.7\% \text{ per year.}$$

31. Since $\ln f(t) = \ln(5e^{1.5t}) = \ln 5 + 1.5t$ we have

$$\frac{d}{dt}\ln f(t) = \frac{d}{dt}(\ln 5 + 1.5t) = 0 + 1.5 = 1.5.$$

35.

(a) $\frac{dB}{dt} = P\left(1+\frac{r}{100}\right)^t \ln\left(1+\frac{r}{100}\right)$. The expression $\frac{dB}{dt}$ tells us how fast the amount of money in the bank is changing with respect to time for fixed initial investment P and interest rate r.

(b) $\frac{dB}{dr} = Pt\left(1+\frac{r}{100}\right)^{t-1}\frac{1}{100}$. The expression $\frac{dB}{dr}$ indicates how fast the amount of money changes with respect to the interest rate r, assuming fixed initial investment P and time t.

36.

The marginal revenue, MR, is obtained by differentiating the total revenue function, R. We use the chain rule so

$$MR = \frac{dR}{dq} = 2000q \cdot \frac{1}{1+1000q^2}.$$

When $q = 10$,

$$\text{Marginal Revenue} = \frac{2000(10)}{1+1000(10)^2} = 0.2 \text{ \$/unit.}$$

37. If the distance $s(t) = 20e^{\frac{t}{2}}$, then the velocity, $v(t)$, is given by

$$v(t) = s'(t) = \left(20e^{\frac{t}{2}}\right)' = \left(\frac{1}{2}\right)\left(20e^{\frac{t}{2}}\right) = 10e^{\frac{t}{2}}.$$

39.

Estimates may vary. From the graphs, we estimate $g(1) \approx 2$, $g'(1) \approx 1$, and $f'(2) \approx 0.8$. Thus, by the chain rule,

$$h'(1) = f'(g(1)) \cdot g'(1) \approx f'(2) \cdot g'(1) \approx 0.8 \cdot 1 = 0.8.$$

43. The chain rule gives

$$\left.\frac{d}{dx}f(g(x))\right|_{x=30} = f'(g(30))g'(30) = f'(55)g'(30) = (1)(\frac{1}{2}) = \frac{1}{2}.$$