Section 3.2

3. $f'(x) = 3x^2 + 3^x \ln 3$

9. $\frac{dy}{dx} = 5(\ln 2)(2^x) - 5.$

17. $P'(t) = 12.41(\ln 0.94)(0.94)^t.$

28. $\frac{dy}{dx} = 2x + 4 - 3/x.$

33. $\frac{f'(t)}{f(t)} = \frac{-144t^{-5}}{35t^{-4}} = -\frac{4}{t}$

37. $y = e^{-2t}$, $y' = -2e^{-2t}$. At $t = 0$, $y = 1$ and $y' = -2$. Thus the tangent line at $(0, 1)$ is $y = -2t + 1$.

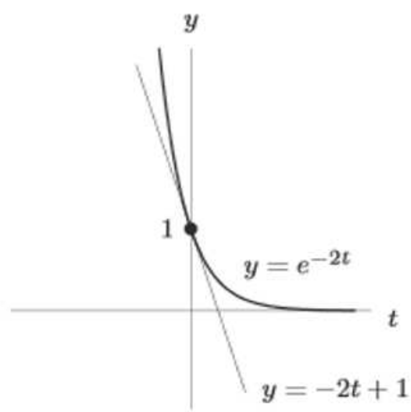

Figure 3.8

46. Since $P = 1 \cdot (1.05)^t$, $\frac{dP}{dt} = \ln(1.05)1.05^t$. When $t = 10$,

$$\frac{dP}{dt} = (\ln 1.05)(1.05)^{10} \approx \$0.07947/\text{year} \approx 7.95¢/\text{year}.$$

49.

$$C(q) = 1000 + 30e^{0.05q}$$
$$C(50) = 1000 + 30e^{2.5} \approx 1365$$

so it costs about \$1365 to produce 50 units.

$$C'(q) = 30(0.05)e^{0.05q} = 1.5e^{0.05q}$$
$$C'(50) = 1.5e^{2.5} \approx 18.27$$

It costs about \$18.27 to produce an additional unit when the production level is 50 units.

51. $C(500) = 1000 + 300\ln(500) \approx 2864.38$; it costs about \$2864 to produce 500 units. $C'(q) = \frac{300}{q}$, $C'(500) = \frac{300}{500} = 0.6$. When the production level is 500, each additional unit costs about \$0.60 to produce.

53.

(a) Since the initial population (at $t = 0$) is 1.166 and the growth rate is 1.5%, we have

$$P = 1.166(1 + 0.015)^t = 1.166(1.015)^t \text{ billion.}$$

(b) Differentiating gives

$$\frac{dP}{dt} = 1.166\frac{d}{dt}(1.015)^t = 1.166(1.015)^t(\ln 1.015).$$

$$\left.\frac{dP}{dt}\right|_{t=0} = 1.166(1.015)^0 \ln 1.015 = 0.017 \text{ billion people per year.}$$

$$\left.\frac{dP}{dt}\right|_{t=25} = 1.166(1.015)^{25} \ln 1.015 = 0.025 \text{ billion people per year.}$$

The derivative $\frac{dP}{dt}$ is the rate of growth of India's population; $\left.\frac{dP}{dt}\right|_{t=0}$ and $\left.\frac{dP}{dt}\right|_{t=25}$ are the rates of growth in the years 2009 and 2034, respectively.