Section 3.1

2. $\frac{dy}{dx} = 0$

5. $y' = 24t^2$

11. $y' = 18x^2 + 8x - 2.$

21. Since $f(x) = \sqrt{\frac{1}{x^3}} = \frac{1}{x^{3/2}} = x^{-3/2}$, we have $f'(x) = -\frac{3}{2}x^{-5/2}$.

28. $y' = 6t - \frac{6}{t^{3/2}} + \frac{2}{t^3}$.

43. Since $f(t) = 700 - 3t^2$, we have $f(5) = 700 - 3(25) = 625$ cm. Since $f'(t) = -6t$, we have $f'(5) = -30$ cm/year. In the year 2010, the sand dune will be 625 cm high and eroding at a rate of 30 centimeters per year.

45. We have $f'(t) = 6t^2$, so the relative rate of change at $t = 4$ is

$$\frac{f'(4)}{f(4)} = \frac{6(4^2)}{2(4^3)+10} = \frac{96}{138} = 0.696 = 69.6\% \text{ per year.}$$

49. $f'(t) = 6t^2 - 8t + 3 \quad$ and $\quad f''(t) = 12t - 8.$

53. To find the equation of a line we need to have a point on the line and its slope. We know that this line is tangent to the curve $f(t) = 6t - t^2$ at $t = 4$. From this we know that both the curve and the line tangent to it will share the same point and the same slope. At $t = 4$, $f(4) = 6(4) - (4)^2 = 24 - 16 = 8$. Thus we have the point $(4, 8)$. To find the slope, we need to find the derivative. The derivative of $f(t)$ is $f'(t) = 6 - 2t$. The slope of the tangent line at $t = 4$ is $f'(4) = 6 - 2(4) = 6 - 8 = -2$. Now that we have a point and the slope, we can find an equation for the tangent line:

$$y = b + mt$$
$$8 = b + (-2)(4)$$
$$b = 16.$$

Thus, $y = -2t + 16$ is the equation for the line tangent to the curve at $t = 4$. See Figure 3.4.

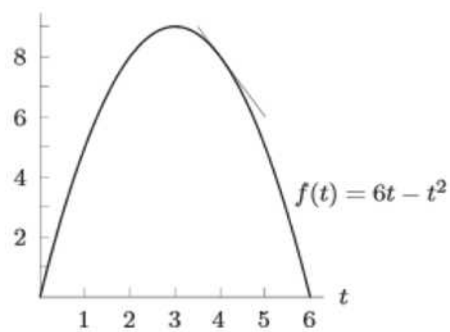

Figure 3.4

61. **(a)** We have $R(p) = pq = p(300 - 3p) = 300p - 3p^2$

(b) Since $R'(p) = 300 - 6p$, we have $R'(10) = 300 - 6 \cdot 10 = 240$. This means that revenues are increasing at a rate of \$240 per dollar of price increase when the price is \$10.

(c) $R'(p) = 300 - 6p$ is positive for $p < 50$ and negative for $p > 50$.

65. **(a)** The marginal cost function equals $C'(q) = 0.08(3q^2) + 75 = 0.24q^2 + 75$.

(b)

$$C(50) = 0.08(50)^3 + 75(50) + 1000 = \$14{,}750.$$

$C(50)$ tells us how much it costs to produce 50 items. From above we can see that the company spends \$14,750 to produce 50 items. The units for $C(q)$ are dollars.

$$C'(50) = 0.24(50)^2 + 75 = \$675 \text{ per item.}$$

$C'(q)$ tells us the approximate change in cost to produce one additional item of product. Thus at $q = 50$ costs will increase by about \$675 for one additional item of product produced. The units are dollars/item.